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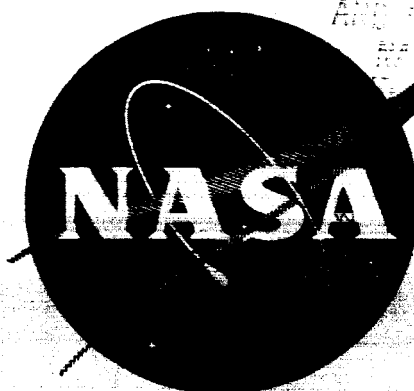
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**DYNAMIC ENVIRONMENTAL TESTING  
AND  
TRANSMISSIBILITY STUDY:**

**P-21**



John Sutton

PREPARED  
BY

[5]

**STRUCTURAL DYNAMICS BRANCH  
TEST AND EVALUATION DIVISION  
OFFICE OF TECHNICAL SERVICES**

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**GODDARD SPACE FLIGHT CENTER,  
GREENBELT, MARYLAND**

DYNAMIC ENVIRONMENTAL TESTING  
AND  
TRANSMISSIBILITY STUDY  
P-21

by  
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SUMMARY

During March, 1961 a redesigned P-21 Ionospheric Probe was vibration tested in order to:

1. Evaluate the new design.
2. Make vibration measurements at critical points where components are installed on the spacecraft.

In addition, a multi-accelerometer force-programming servo control system for use in vibration testing the P-21 spacecrafts was used for the first time, and evaluated.

A brief study was made of some of the data analysis problems which the Structural Dynamics Branch will soon face as a result of increased test requirements.

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## INTRODUCTION

During the summer of 1960, a type P-21 Ionospheric Probe was vibration tested and found to be unsatisfactory. The original design was modified extensively in an attempt to protect the delicate electronic components from damaging structural vibration. The spacecraft electronics consist of two major items: An RF-propagation experiment transmitter and an electro-acoustic probe.

On February 24, 1961 and March 2 and 3, 1961, the redesigned Prototype P-21 was vibration tested on the GSFC MB Electronics Model C-50 vibration exciter system. This test served several purposes: To evaluate the remodeled spacecraft, to make vibratory acceleration measurements at critical points on the spacecraft, to evaluate a multi-accelerometer feedback vibration control system exciter, and to explore some of the data reduction and analysis problems the Test and Evaluation Division must deal with in the near future.

## APPARATUS:

The vibration equipment employed during the dynamic testing of the P-21 Prototype in February and March 2 and 3, 1961 consisted of an MB Electronics Model C-50 vibration exciter driven by a Model T-888 power amplifier. The control unit was an MB Model T288 complex motion console. Vibration data was sensed by Endevco Model 2221 C and Model 2702 amplifiers and recorded simultaneously on a Honeywell Model 3171-4A1 multi-channel frequency-modulated tape recorder. A modified Ling Electronics audio frequency mixer, four Krohn-Heit Model 330-M bandpass filters, and a Tektronix Model 535 oscilloscope were also required for an evaluation of a special force programming system. Figures (33) through (38) are photographs of the data recording and force programming systems, the Model C-50 vibration exciter, and the oil film horizontal vibration facility.

## PROCEDURE

On February 24, 1961, the P-21 Prototype spacecraft was mounted on the horizontal slip table test fixture (figures 37 and 38) and vibration tested with a sinusoidal input acceleration of slowly increasing frequency. The orientation was such that the first vibration axis corresponded approximately with antenna HF #2. The sinusoidal test was followed by a random noise vibration test. The spacecraft was rotated 90° and the above mentioned tests were repeated in reverse order. The 600 cps, X-248 motor resonant burning simulation test was run first in the second transverse axis and then with the spacecraft in the original orientation. On March 2, 1961, the spacecraft was mounted on the force testing fixture (visible behind the filters in Figure 33) and the thrust axis noise vibration was applied to the payload. The test was stopped until March 3, 1961, due to payload transmitter failure during this random noise test. The transmitter was repaired, after which the sweep-frequency sinusoidal, random, and 600 cps tests were completed. The following is a tabulation of the sinusoidal testing levels employed:

<u>Direction</u>	<u>Frequency Range</u> <u>cps</u>	<u>Test Duration</u> <u>Min.</u>	<u>Acceleration</u> <u>g 0-to-Peak</u>
Thrust (z-z Axis)	5-50	1.6	2.3
	50-500	1.6	10.7
	500-2000	1.0	21.0
	2000-3000	0.26	54.0
Lateral A (x-x Axis)	5-50	1.6	0.9
	50-500	1.6	2.1
	500-2000	1.0	4.2
	2000-3000	0.6	17
Lateral B (y-y Axis)	5-50	1.6	0.9
	50-500	1.6	2.1
	500-2000	1.6	4.2
	2000-3000	0.6	17

During the sweep-frequency sinusoidal tests, the vibration testing was stopped momentarily at 500 cps to switch servo control ranges. Also, a high frequency limitation of the vibration exciter of 3000 cps was not exceeded, though the specification requires the test to be carried out to 5000 cps.

The random motion input, except for equalizer response limitations, was essentially as specified:

<u>Direction</u>	<u>Frequency Range</u> cps	<u>Test Duration</u> Min.	<u>PSD</u> <u>Level</u> <u>g<sup>2</sup>/cps</u>	<u>Approx.</u> <u>Accel.</u> <u>g-rms</u>
Thrust	5-200	4 (total)	0.12	5.0
	200-400		12 db/	2.4
	400-2000		octave	4.0
			roll-off 0.01	
Lateral A	5-25	4 (total)	0.60	3.5
	25-100		12 db/	1.7
	100-2000		octave	4.1
			roll-off 0.01	
Lateral B (y-y Axis)	5-25	4 (total)	0.60	3.5
	25-100		12 db/	1.7
	100-2000		octave	4.1
			roll-off 0.01	

#### COMBUSTION RESONANCE VIBRATION

<u>Direction</u>	<u>Frequency</u> cps	<u>Vector</u> <u>Acceleration g</u>	<u>Duration Sec.</u>
Thrust Axis	550-650	120	30
Transverse Axis	550-650	21	30 each axis

The "equalizer limitations" mentioned above are readily seen in Figures 23, 24, 25, and 29. The equalization is everywhere within a few db of specifications. From Figures 5, 6, 7, and 8, it is evident that the 600 cps resonant burning simulation test was shorter than specified.

Because a suitable horizontal force programming fixture has not yet been fabricated, it was necessary to assume that the payload had an apparent mass of five pounds for the lateral 600 cps testing. The 600 cps thrust axis vibration servo signal was derived, as it was for the entire vibration program, from a single accelerometer ... but the vibration exciter input was varied "up" and "down" at the command of an observer who was monitoring, via strain gages, 2702 amplifier, and a meter, the 600 pound force input to the payload. The control accelerometer was located at the top of the force fixture in one of the notches visible in Figure 33. During the other portions of the testing program for the P-21 Prototype, the control accelerometer was located on the spacecraft fixture.

## DISCUSSION

### Multi-accelerometer Servo Control:

The fixture employed in vibration testing Argo D-4 spacecraft is quite flexible at the high frequencies (above 1,000 cps). There is relative motion between the exciter table and the spacecraft interface and between any two points on the spacecraft interface which contact the exciter fixture mounting plate. Because it is desired to apply a known sinusoidal motion to the base of the spacecraft, and because a single servo control accelerometer will not necessarily "see" an acceleration representative of the axial input to the spacecraft regardless of location at the base of the spacecraft, it seems advisable that a better method of obtaining the servo control signal should be found. Ideally, many accelerometers could be evenly spaced on the circumference of the spacecraft interface, the output signals averaged, and the resulting signal fed back into the existing exciter servo loop. A practical, economical compromise was necessary, however, because each accelerometer signal must be filtered separately. Accordingly, for the servo system evaluation test three accelerometers were installed 120° apart on the top of the test fixture and connected via appropriate amplifiers, filters, and an adder network to the vibration exciter servo control system. Separate filtering is necessary because rattling of spacecraft antennae, springs, etc. can generate high frequency vibration within the spacecraft structure. These vibrations are reflected back to the base of the spacecraft where they are sensed by the control accelerometers. Two such rattling signals could add to form lower frequency "beats" which would pass through a single filter at the output of the mixer and thereby increase the total feedback signal, resulting in an "undertest". By passing each accelerometer signal through a filter of cutoff frequency as low as possible consistent with the highest test frequency encountered, one can eliminate most of the "beats" and thus eliminate the undertesting. A detailed description of a signal adder circuit suitable for a multi-accelerometer servo control system is presented in Enclosure 1.

### Force Programming:

The 550-650 cps sweep test is executed at an acceleration level which is related to the apparent weight of the spacecraft. What is ultimately desired is a vibratory input force of 600 pounds rather than any particular acceleration. In order to obtain a servo control signal to facilitate programming an input force, high sensitivity strain gages were mounted on a special support fixture. The fixture consists of an aluminum hoop machined to a thickness of 0.050 inch at the center. The design natural frequency with a load of 5 pounds is 2400 cps. It was estimated that the mass of the portion of the strain gage fixture above the strain gages is 3 pounds. Thus the strain gages sense the input force applied to the spacecraft plus the force required to vibrate the 3 pound portion of the fixture at the acceleration level measured on the top of the fixture. Because the force fixture is nonresonant at the test frequency, the signal proportional to the 600 pound force input to the spacecraft at 600 cps can be obtained by subtracting the instantaneous accelerometer signal from the instantaneous strain gage signal.

The subtraction of the instantaneous amplitudes of the acceleration and force signals can readily be done by an electronic "adder" such as described in Enclosure (1) provided an additional phase inverter stage is connected to one input. An oscilloscope such as the Tektronix model 535 with a differential input pre-amplifier unit also lends itself readily to this application and provides a high quality signal amplifier and indicating device as well. The three-accelerometer system previously described could be used to supply an acceleration signal to the force-acceleration adder. Details of a complete force-control servo system are included in Enclosure (2).

The force programming described here is a first attempt to more accurately simulate the true dynamic environment of a spacecraft. It is one step in the direction of true impedance testing and by no means is the best possible test procedure. The concept of mechanical impedance is important because the payload vibration environment is affected by the impedance of the supporting launch vehicle structure in such a way that actual field conditions include simultaneous "rocking" and torsional vibration of the spacecraft in addition to the axial vibration. The Argo D-4 vibration specification requires lateral vibration in an attempt to simulate this complex motion in the form of an easily reproducible test. Neither the uncontrolled "rocking" of the spacecraft on the test fixture, nor the lateral vibration test is a perfect simulation of the in-flight environment. The Structural Dynamics Branch, therefore, may attempt to more adequately simulate launch vehicle environments through the use of a multi-exciter system. Such a system could, theoretically, be programmed with a measured launch vehicle impedance. The approximation of the true environment via a three dimensional system such as this might require extensive mathematical treatment, an electronic computer, and unusual impedance programming circuitry.

#### Data Reduction and Analysis:

On February 20, 1961 the Minneapolis Honeywell Company loaned an FM tape recorder to the Structural Dynamics Branch for evaluation purposes. This machine was exploited with the purpose of testing the data handling capabilities of a tape recording system. The machine proved immediately to be a great asset. Acceleration and force signals were recorded during the actual vibration testing. Later the signals were re-recorded on x-y plotters via random signal analyzers and logarithmic converters. The sinusoidal signal frequency could have been accurately scaled on the abscissas of the resulting charts, but there was no time to eliminate the ground loop which resulted when the tape recorder was connected to the frequency analog signal from the exciter system oscillator dial. Frequencies were measured with an electronic counter at some points during tape playback but it was not possible, even with the aid of a low pass filter, to obtain many frequency calibration points because of the noisy character of much of the recorded data. This noise resulted from rattling of springs, antennae, etc. and also contributed to the average amplitudes of the signals recorded.

The random signals were played back into an 80 channel analyzer-filter system. This system is discussed in Enclosure (5). A comparison can be made of Figures (11), (23), (24), and (25). Figure (23) is an 8 minute analysis, made with a Technical Products model TP 627 wave analyzer, of a 2000 cycle bandwidth random signal. A 5 cycle bandwidth filter was employed. Figures (24) and (25) were made at different speeds with an 80 channel filter analyzer. In Figure (11) an 80 second plot made with this analyzer is compared with a 200 second plot. Figures (24) and (25) correspond within 1 db at all frequencies. They are in agreement except for unessential detail revealed by the Technical Products Analyzer. The excessive "noisiness" of the Technical Products Analyzer record is due to the fact that the averaging time of the filter output was shorter than the optimum value.

These comparisons indicate that the 80 second, 80 channel filter analyzer graphs are, for all practical purposes, as good as the graphs obtained in 8 minutes via the Technical Products sweep-frequency-filter system. As is discussed in Enclosure (5), the output voltages from the 80 narrow filters are approximately Gaussian. Hence, these signals are proportional to acceleration spectral density without the usual squaring operation. The 80 channel analyzer enjoys another advantage over the Technical Products analyzer in that one can scale off frequencies on the graphs simply by counting the number of filter response peaks from one edge. (The filters are 25 cps wide and have center frequencies of 25 cps, 50, 75, ...) The two types of filters discussed here have some problems in common in that the random signal to be analyzed is usually recorded on a tape loop and played back continuously on a tape recorder. One of the important problems encountered in continuous random signal analysis with a tape loop system, that of selecting the minimum tape loop length consistent with usable accuracy, is discussed in Enclosure (4).

A description of the "quality" of an accelerometer signal is difficult to present on an x-y recorder. A fundamental sinusoidal waveform can often be recovered from a "noisy" signal via filters, subsequently measured with respect to amplitude and phase, and compared with other similarly processed voltages. In like fashion harmonic components can be selectively filtered and measured. Obviously, such analysis would require an inordinate quantity of expensive electronic equipment and many valuable man-hours. Perhaps a simpler solution to this analysis problem is that recently developed by the Chadwick-Helmuth Company. With the aid of a modification of the "Slip-Sync" idea, it is possible to reproduce galvanometer signals in slow motion, preserving original amplitudes and phases in the final recording. In this manner, ten inch long sections of "slow motion" chart recordings can be made of important information and enclosed in analysis reports without the necessity of using large quantities of expensive chart paper. (The Structural Dynamics Branch recently placed a \$5000 order for 100 rolls of such paper).

### Random Motion Test Specification:

The fundamental relationship between acceleration and acceleration spectral density is:

$$a^2 = \int_{f_1}^{f_2} G(f) df$$

where  $G(f)$  is the spectral density as a function of frequency. The test specification quoted in Enclosure (6) requires a 24db per octave rolloff between 200 and 400 cps. The spectral density function which fulfills this requirement is:

$$G_{f1} \times (f1)^4 \times f^{-4}$$

If we solve the fundamental equation for the case of the x-x axis random motion specification, we have:

$$\begin{aligned} a^2 &= 0.06 \times (200)^4 \times f^{-4} \\ a^2 &= .06 \times 16 \times 10^8 \left[ \frac{-1}{3(400)^3} - \frac{-1}{3(200)^3} \right] \\ &= .06 \times 16 \times 10^8 \times 10^{-6} \left[ \frac{1}{24} - \frac{1}{192} \right] = 3.51 \end{aligned}$$

$a = 1.87$  g rms. This number is slightly greater than the 1.7g rms quoted in the specification and was the cause of some confusion until it was discovered that the specification was written for a particular filter. The Krohn-Heit Model 330-M has a voltage rolloff characteristic of approximately 24 db per octave and will meet the specification requirements within  $\pm 3$  db. A literal 24 db per octave characteristic would require that the acceleration spectral density at 400 cps be  $0.004 \text{ g}^2/\text{cps}$  rather than the  $0.005 \text{ g}^2/\text{cps}$  specified.

### Aural Analysis:

There is one method of data analysis which seems not to have received enough attention and which may soon be put to use by the Structural Dynamics Branch: It is possible to play a tape recording of an accelerometer signal through an audio amplifier and loudspeaker and to listen to the resulting sound. This process must be experienced to be fully appreciated. It facilitates the analysis of events which occur during a sinusoidal test, i.e. antennae rattling, etc., but the most important application is in the analysis of a random signal. An oscilloscope presentation of a random signal will not synchronise in any meaningful fashion, nor will a continuous oscillograph record necessarily reveal to any but the most experienced observer much of the hidden detail. It is remarkably simple, however, to



obtain information aurally. Rattling of parts and components can be heard above the noise signal and localized by successive monitoring of several accelerometers mounted at various points on the structure.

Aural monitoring was vividly demonstrated in October of 1960 when Dr. R. O. Belsheim of the Naval Research Laboratory played a recording of a signal from an accelerometer mounted on the X-248 rocket motor. The 600 cps resonant burning was clearly distinguishable as were the sounds of portions of the motor liner striking the throat of the nozzle.

## RESULTS:

### Vibration:

On February 24, 1961 after the two complete vibration programs had been run in the transverse axes, the payload was mounted in position for the thrust axis vibration. After the lunch recess, it was discovered that the counter employed to monitor the payload transmitter frequency indicated that the frequency had shifted slightly. Further testing was delayed while the payload and the counter were both removed to Building II for examination. The counter proved to be accurate, but the frequency shift was not judged to be a failure because the transmitter had stabilized at a new frequency.

Vibration testing was continued again on March 2, 1961 with the thrust axis random test. After three minutes of vibration the payload transmitter failed and the test was discontinued, pending examination. Subsequently, a broken wire in the transmitter module was replaced and testing was resumed on March 3, 1961, beginning with the sweep frequency sinusoidal test. After about three minutes of sinusoidal vibration, the spacecraft transmitter frequency had drifted about 100 cps, but then stabilized at the new frequency. The testing program was completed without further mishap.

### Multi-accelerometer Servo:

With regard to the multi-accelerometer mixing problem, a successful bare-table sinusoidal sweep frequency test was completed using the mixer output signal for the servo control voltage source. Time did not permit comparison of the mixer signal with single accelerometer signals to verify the cancellation of rocking modes; nor was there time to test the theoretical operation of the complete force programming circuit prior to spacecraft vibration testing. Obviously, these unproved systems could not be used during the actual testing of the P-21 spacecraft.

### Data Reduction:

The data reduction problem-study revealed that probably the most practical way of handling vibration data, despite the dynamic range limitation (approximately 40 db for the Honeywell machine), is to record the data on a tape recorder and then to transfer this data to x-y recorders.

The sinusoidal test recording should be supplemented by a d. c. frequency analog voltage in order that the x axis of the resulting curves can be calibrated with respect to frequency. If it is desired to plot relative responses at various points on a spacecraft with respect to an input, i.e., "Q", it would be well to have a voltage ratio computer such as is described in Figure (39). Such a computer would save a considerable amount of time over the point-by-point hand-plotting method, and would be more accurate by virtue of continuity. Alternatively, the Moseley Company has developed a circuit wherein two logarithmic converters and an impedance network perform the same function as a ratio computer. An arrangement of two or more x-y plotters could be set up to reduce data from several tape channels simultaneously, thereby reducing analysis time by a factor of 2 or 3 ... etc.

#### Acceleration Measurements:

The data reduction method described above was developed to present in a more convenient form the requested information obtained from the vibration level measurements made on the spacecraft. Because it was desired to know what vibratory accelerations a replacement component of approximately the same dimensions as the Acoustic Probe would "see" on top of the spacecraft frustum, it was not necessary to plot "Q" versus frequency. Had graphs of transmissibility been required, a ratio computer would have been a prerequisite. The measurement data is presented in Figures 1-8 and 15-32 for accelerometers on the top of the frustum as well as those cemented on top of the transmitter and on the telemetry base plate. The relatively large number of graphs of useful data enclosed, far more than it has formerly been possible to present, illustrates the value of the tape-analyzer-plotter system. Figures #9, 10, 12-22, 26-28, 30-32 are enclosed as requested. They may be referred to for comparison in future reports. The graphs were produced in 1 1/2 man days, a saving of approximately 5 man days over conventional methods. "Q" values of the maximum sinusoidal responses and of the overall random response are tabulated below. Frequencies above 1000 cps are not considered because of errors introduced by the flexibility of the mounting fixture.

# P-21 Payload Maximum Sinusoidal Response

## (1) THRUST AXIS:

<u>Input, g</u>	<u>Frequency, cps</u>	<u>Frustrum</u>		<u>Transmitter</u>		<u>Base Plate</u>	
		<u>g.</u>	<u>Q</u>	<u>g</u>	<u>Q</u>	<u>g</u>	<u>Q</u>
14	200	45	3.2	28	2.0	140	10
19	600	25	1.3	25	1.3	63	3.3

## (2) TRANSVERSE:

<u>Input, g.</u>	<u>Frequency, cps</u>	<u>Transmitter, g</u>		<u>Transmitter, g</u>	
		<u>1st Orientation</u>	<u>Q</u>	<u>2nd Orientation</u>	<u>Q</u>
2.1	90	10	4.8	-	-
2.1	100	-	-	12.6	6.0
2.1	200	10	4.8	4.5	2.1
2.1	400	10	4.8	-	-
4.2	600	10	2.4	-	-
4.2	650	-	-	7.1	1.7

## THRUST AXIS RANDOM

	<u>Acceleration</u>	
	<u>g rms</u>	<u>"Q"</u>
Exciter Table	2.4	-
Transmitter	4.3	1.8
Base Plate	4.8	2.0
Frustrum	4.8	2.0

## CONCLUSION:

### Vibration:

Testing of the P-21 Prototype in February and March 1961, has revealed that a substantial improvement in design was made over a previous model tested during 1960. The fact that the transmitter failed once during testing indicates that there may still be some packaging problems to solve.

### Force Programming:

No definite conclusions can yet be reached concerning the practicality of force programming; however, preliminary tests indicate that the system merits further investigation.

### Data Reduction:

The data reduction methods developed show considerable promise and when circuit details are completed, should prove extremely valuable.

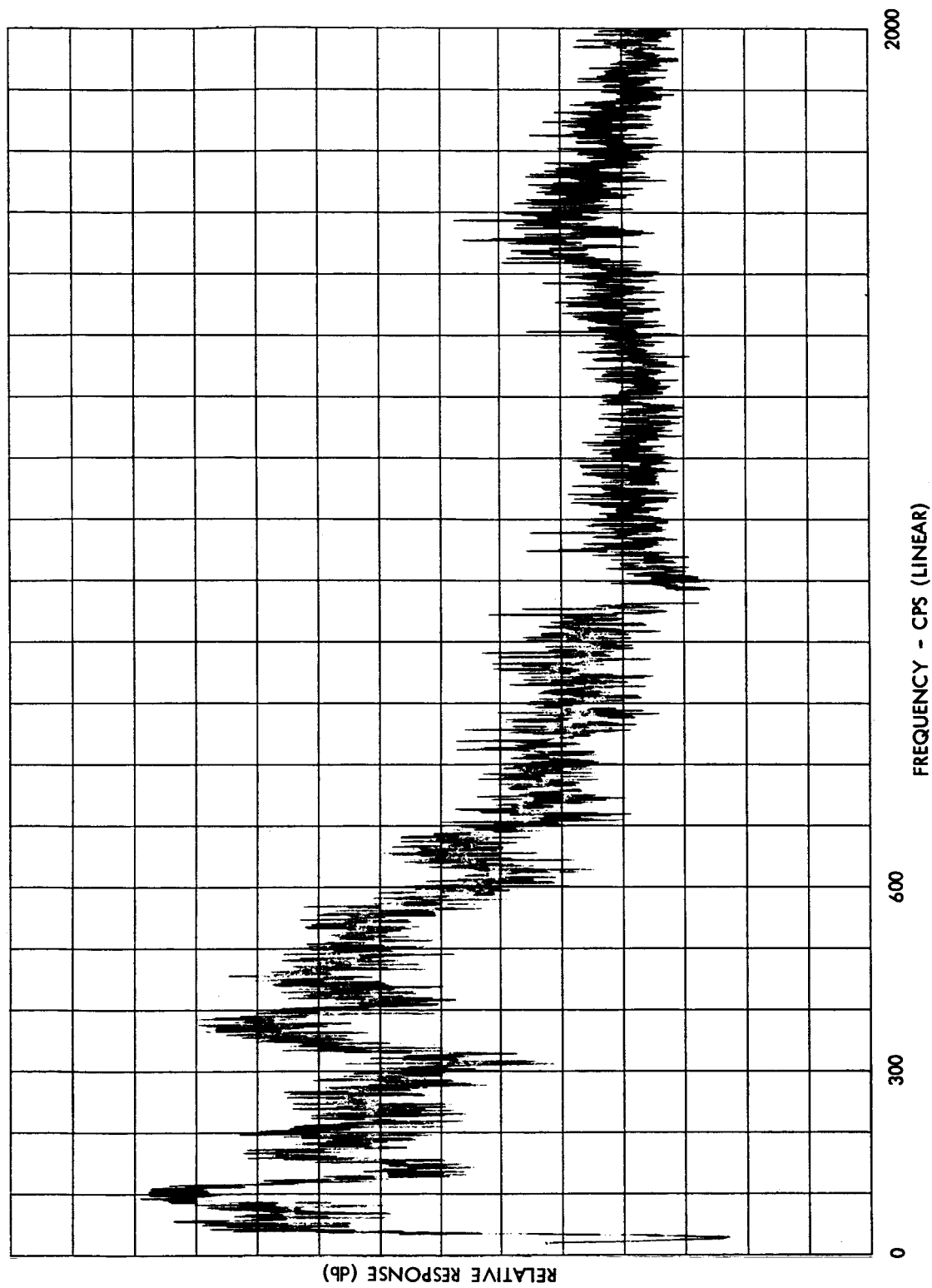


Figure 1. P-21 Random Vibration Test of 24 February 1961  
Vibration Axis: First Orientation Transverse  
Accelerometer Location: Top of Payload Transmitter  
Scale Factor: 10 db/inch  
TP Analyzer: 5 cps Filter, 8 min.

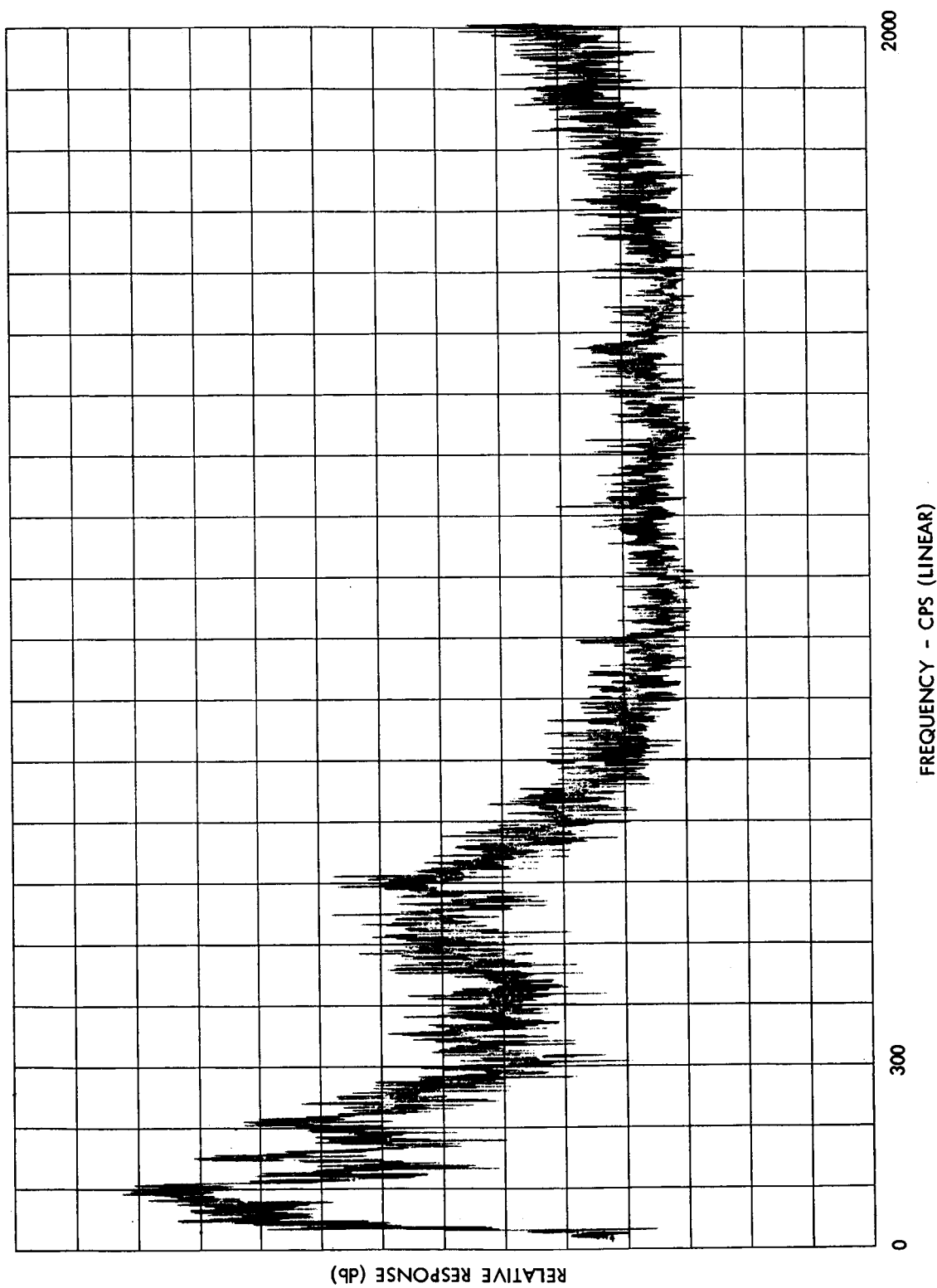


Figure 2. P-21 Random Vibration Test of 24 February 1961  
Vibration Axis: Second Orientation Transverse  
Accelerometer Location: Top of Payload Transmitter  
Scale Factor: 10 db/inch  
TP Analyzer: 5 cps Filter, 8 min

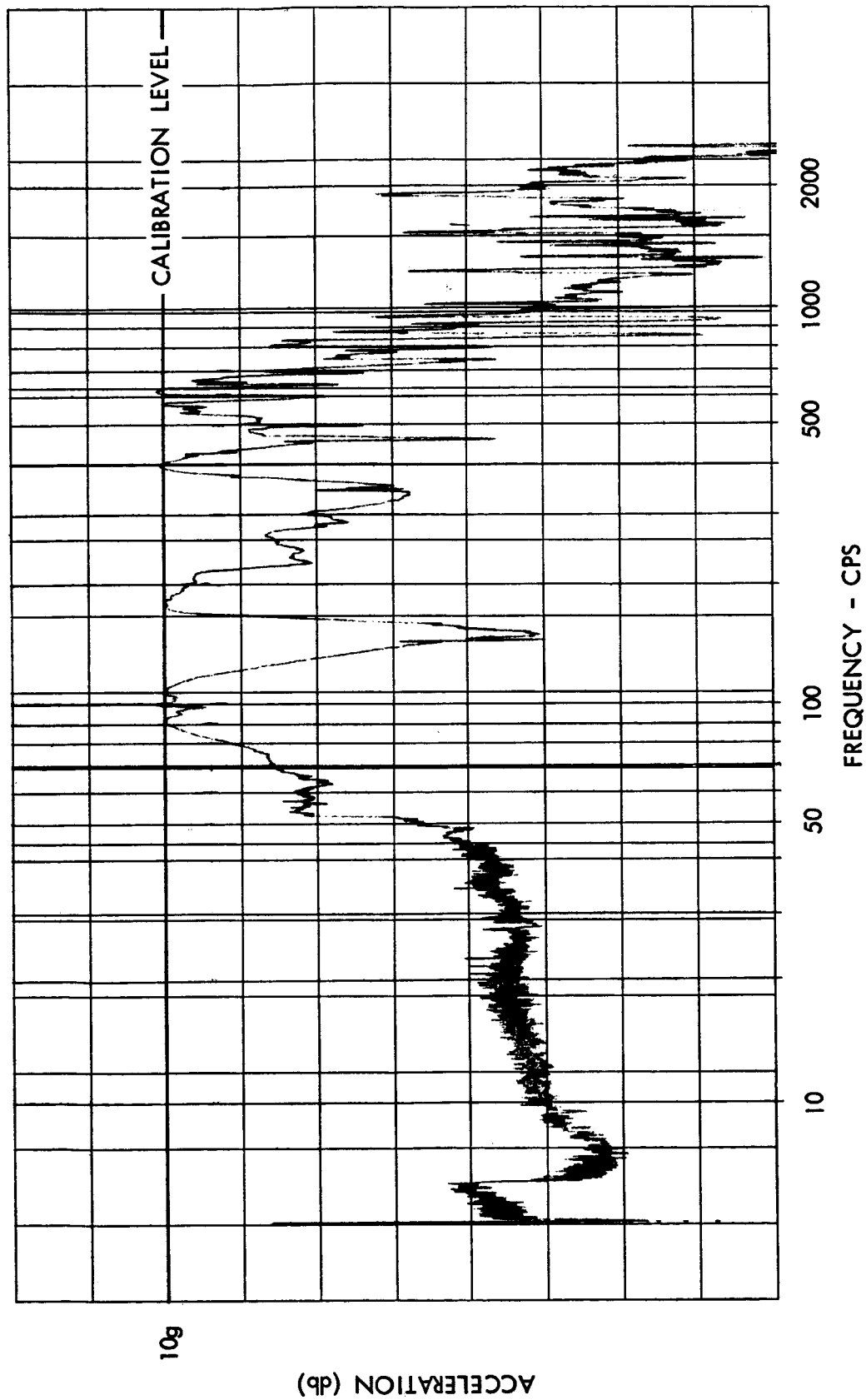


Figure 3. P-21 Test of 24 February 1961 Sinusoidal Sweep  
Vibration Axis: First Orientation Transverse  
Accelerometer Location: Top of Payload Transmitter  
Scale Factor: 10 db/inch

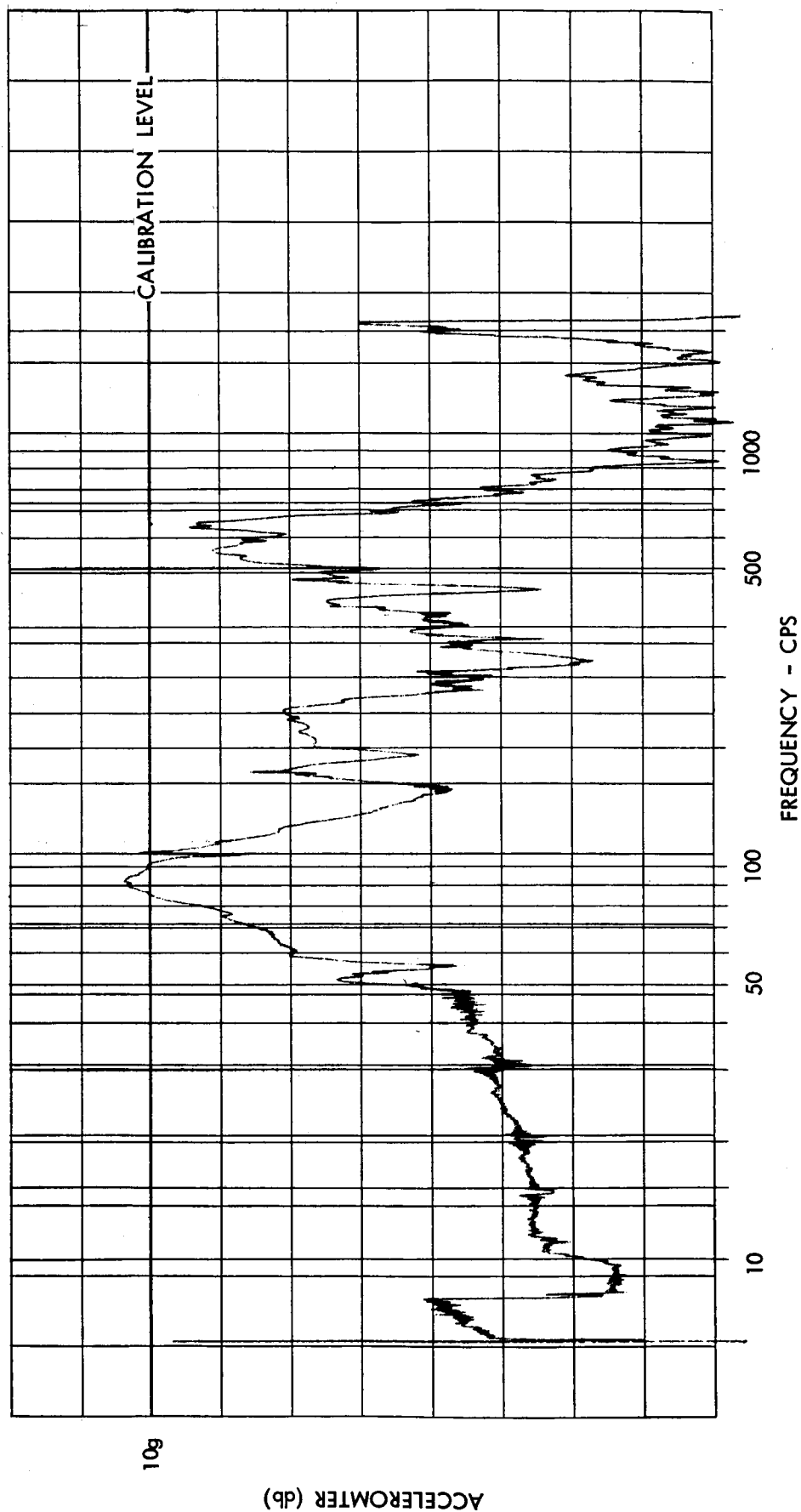


Figure 4. P-21 Test of 24 February 1961 Sinusoidal Sweep  
Vibration Axis: Second Orientation Transverse  
Accelerometer Location: Top of Payload Transmitter  
Scale Factor: 10 db/inch



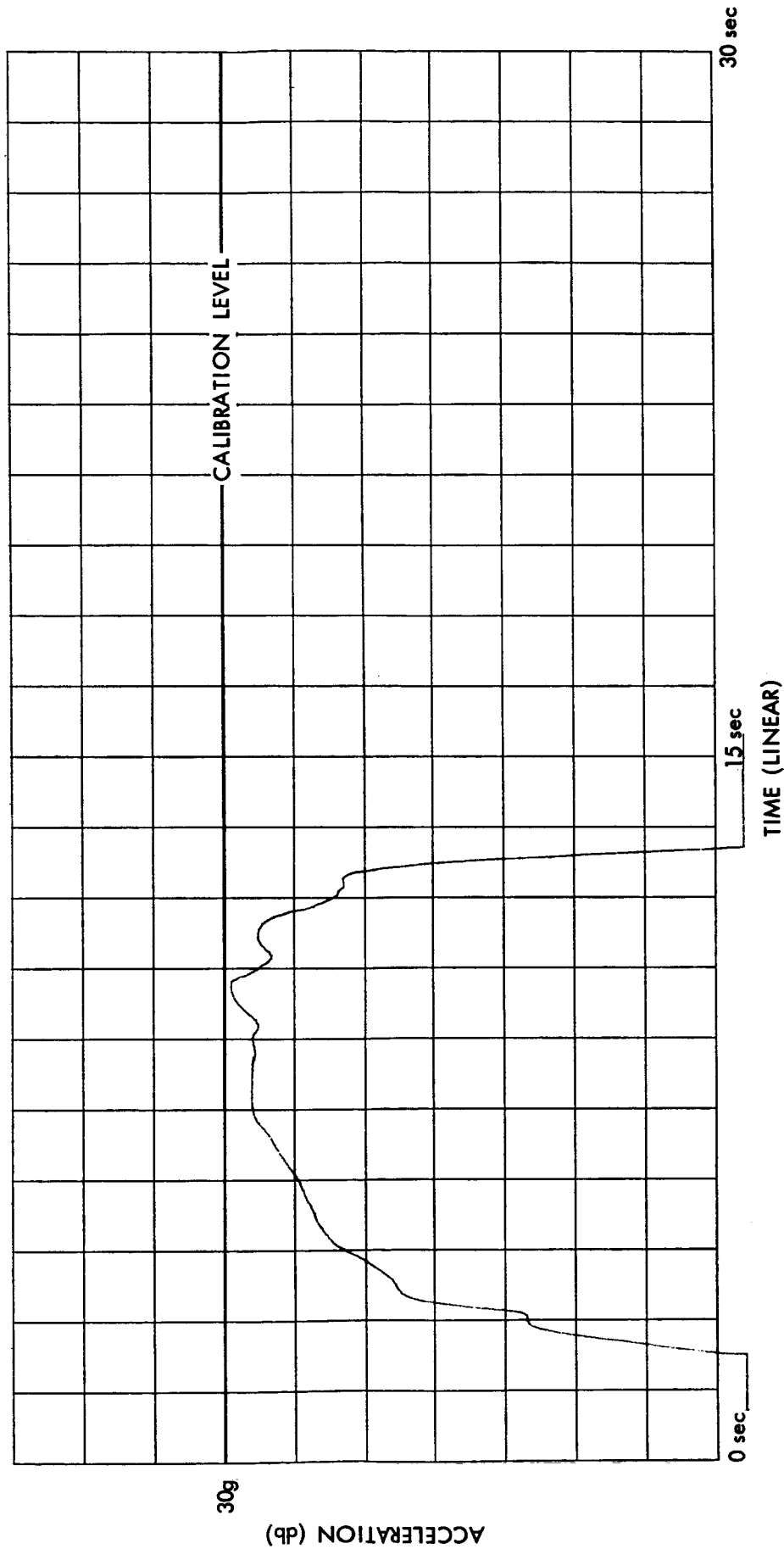


Figure 5. P-21 Test 24 February 1961 550-650 cps Sweep  
Vibration Axis: First Orientation Transverse  
Accelerometer Location: Top of Payload Transmitter  
Scale Factor: 10 db/inch

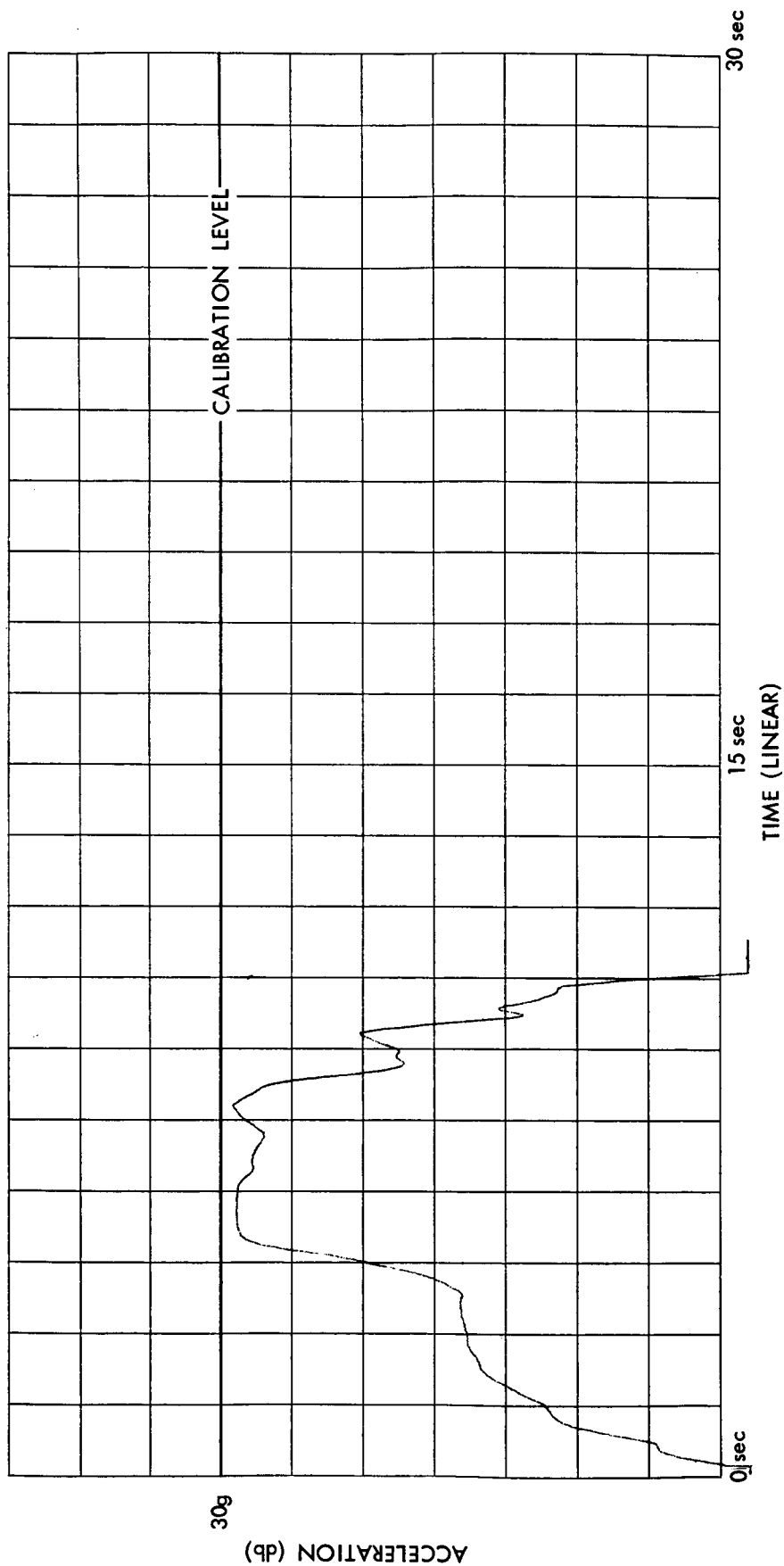


Figure 6. P-21 Test of 24 February 1961 550-650  
Cross Axis: First Orientation Transverse  
Accelerometer Location: Top of Payload Transmitter  
Scale Factor: 10 db/inch

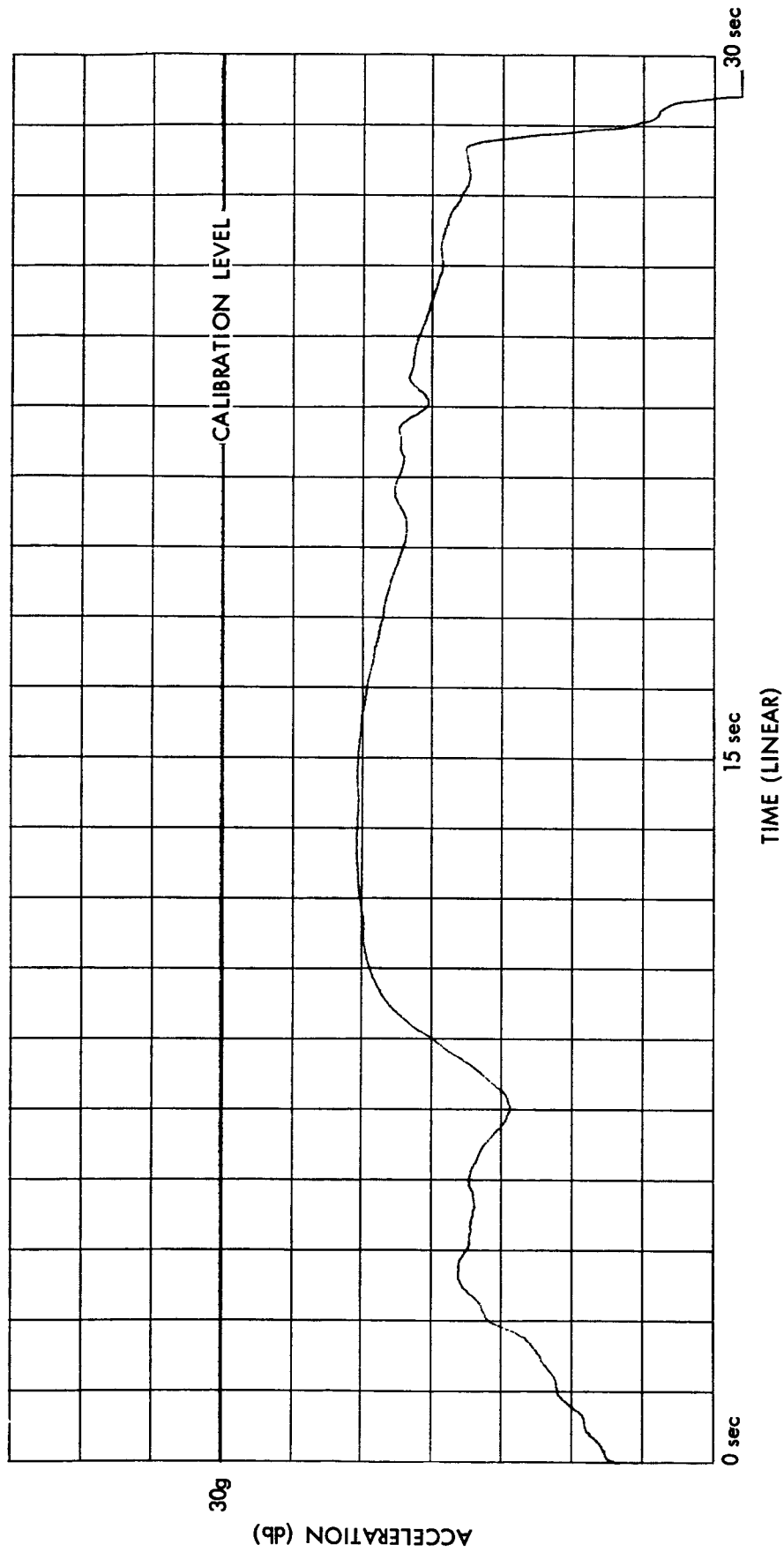


Figure 7. P-21 Test of 24 February 1961 550-650 cps Sweep  
Vibration Axis: Second Orientation Transverse  
Accelerometer Location: Top of Payload Transmitter  
Scale Factor: 10 db/inch

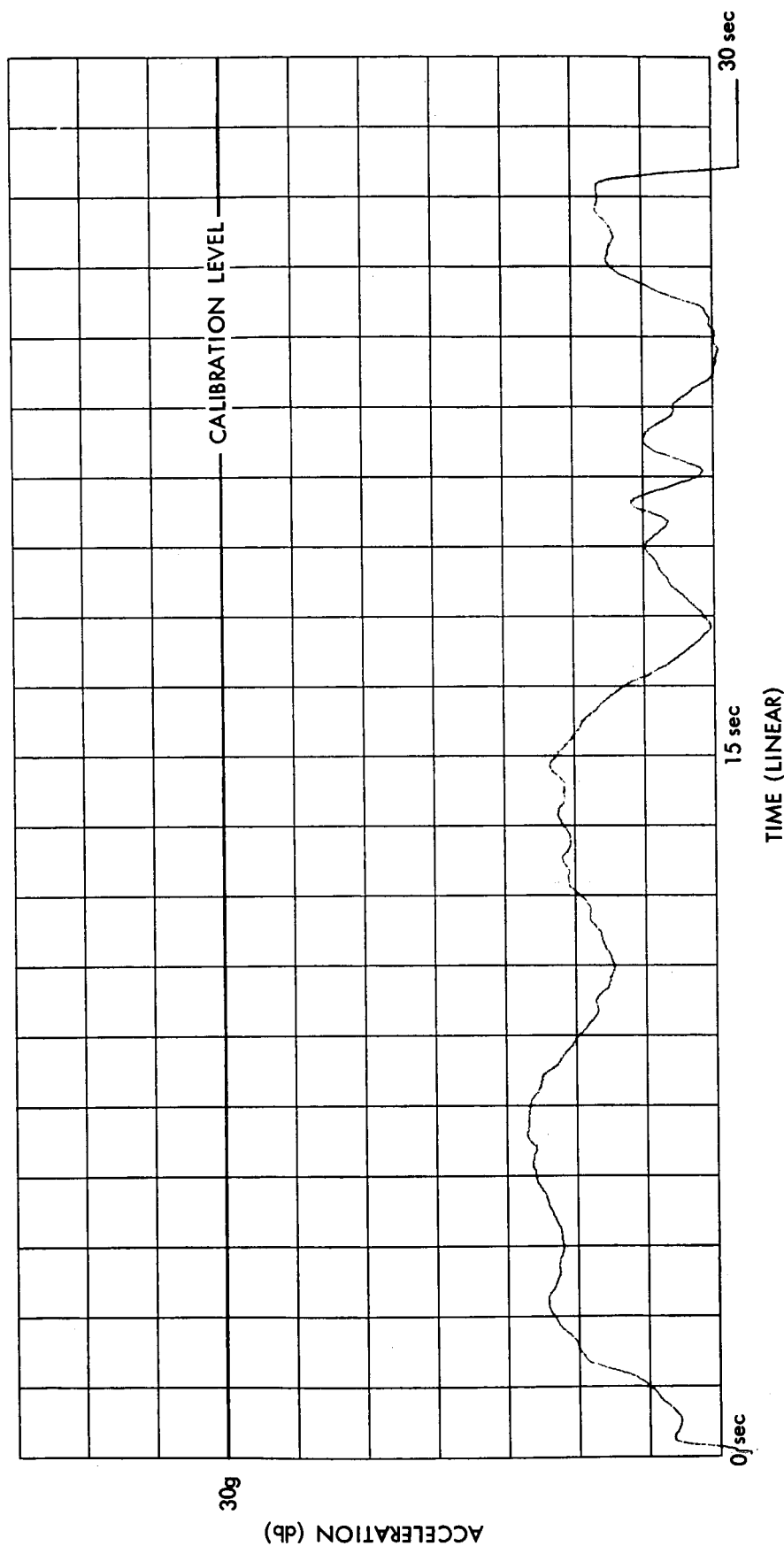
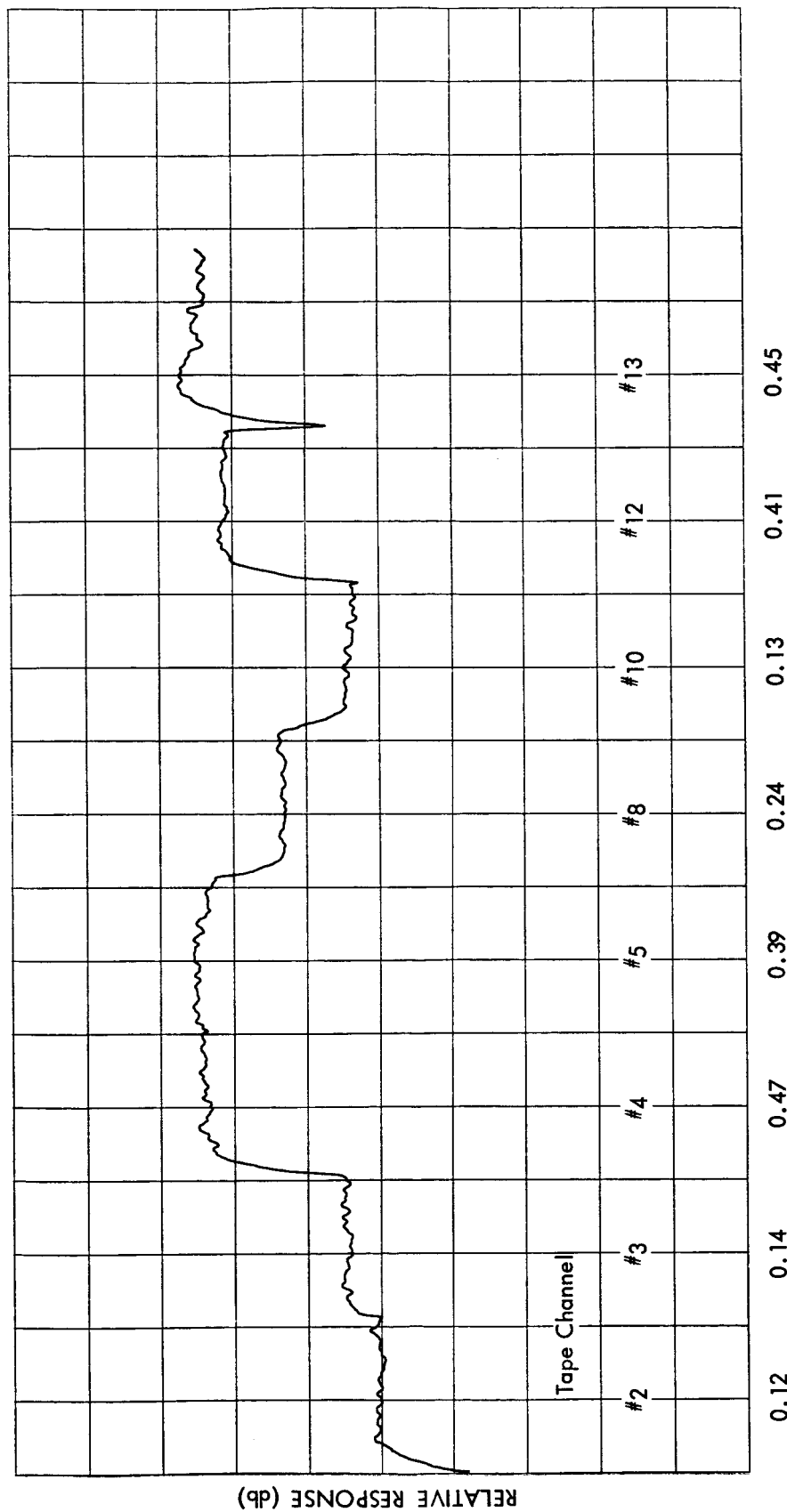


Figure 8. P-21 Test of 24 February 1961 550-650 cps Sweep  
 Cross Axis: Second Orientation Transverse  
 Accelerometer Location: Top of Payload Transmitter  
 Scale Factor: 10 db/inch



TRUE RMS VOLTAGE, DECIMAL  
PARTS OF (1 volt = cal on ch 8)

Figure 9. Relative Signal Levels From Eight  
Data Channels: P-21 Random Vibration  
Test of 3 March 1961, Thrust Axis.

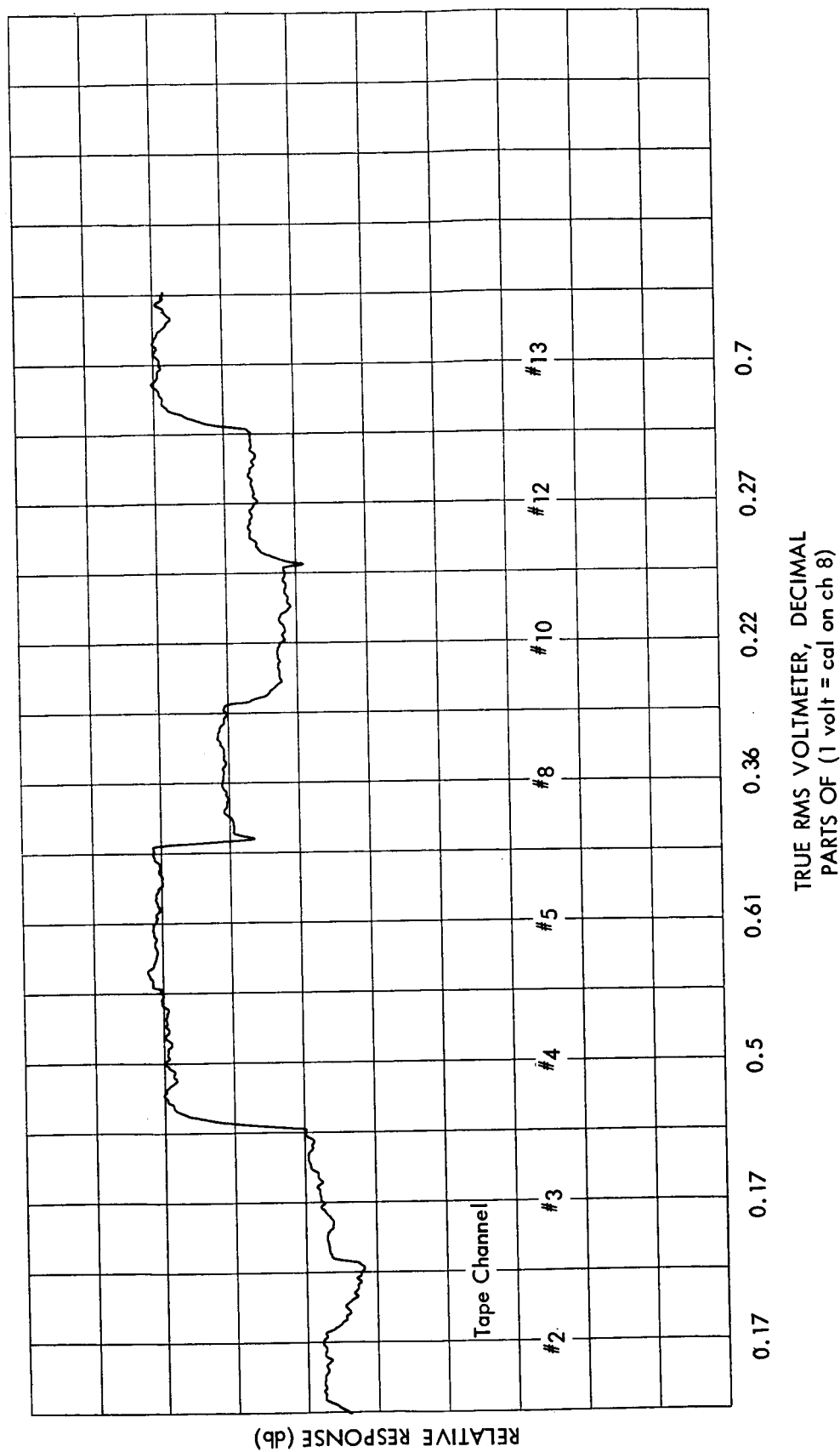


Figure 10. Relative Signal Levels From Eight Data Channels: P-21 Random Vibration Test of 2 March 1961, Thrust Axis.

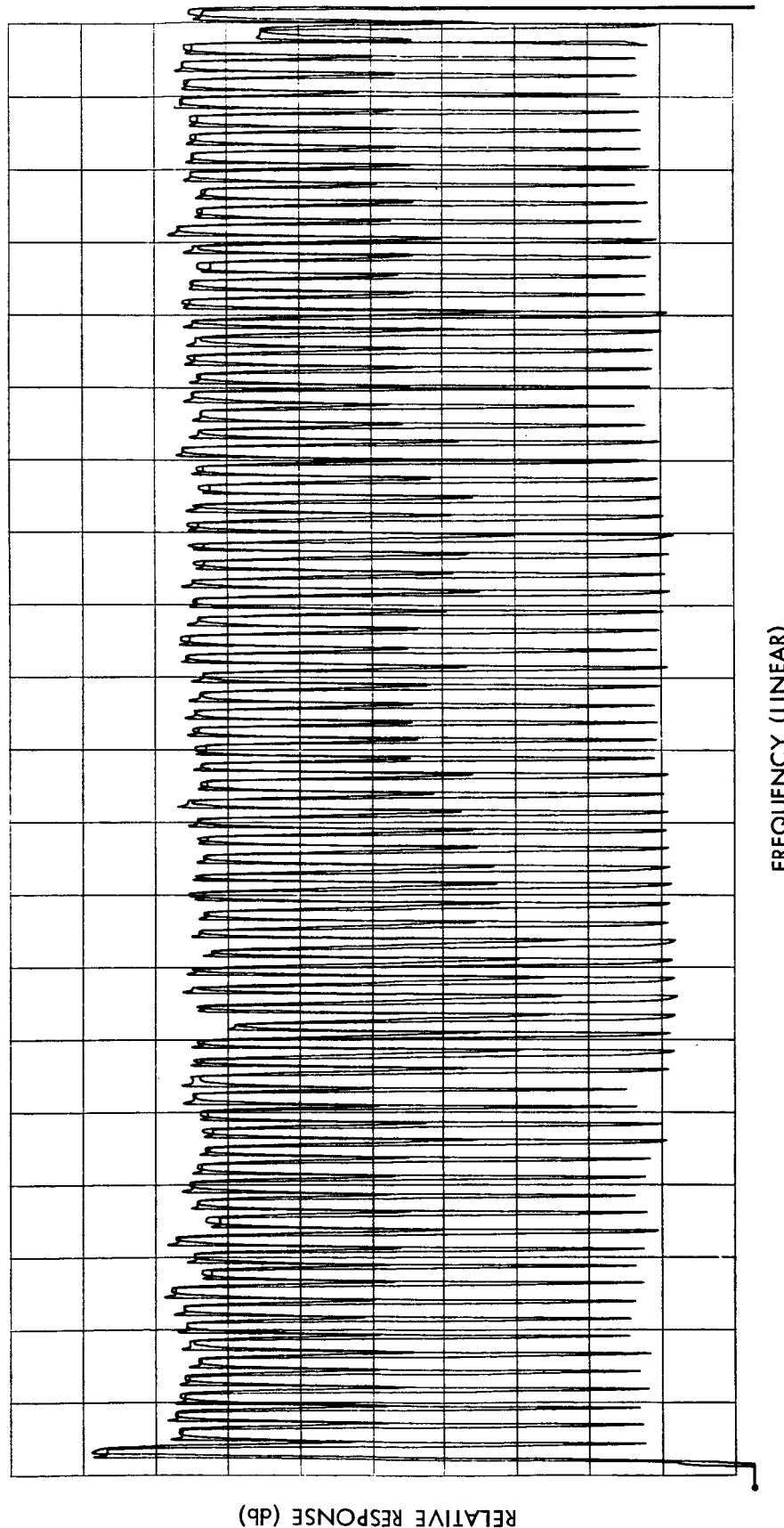
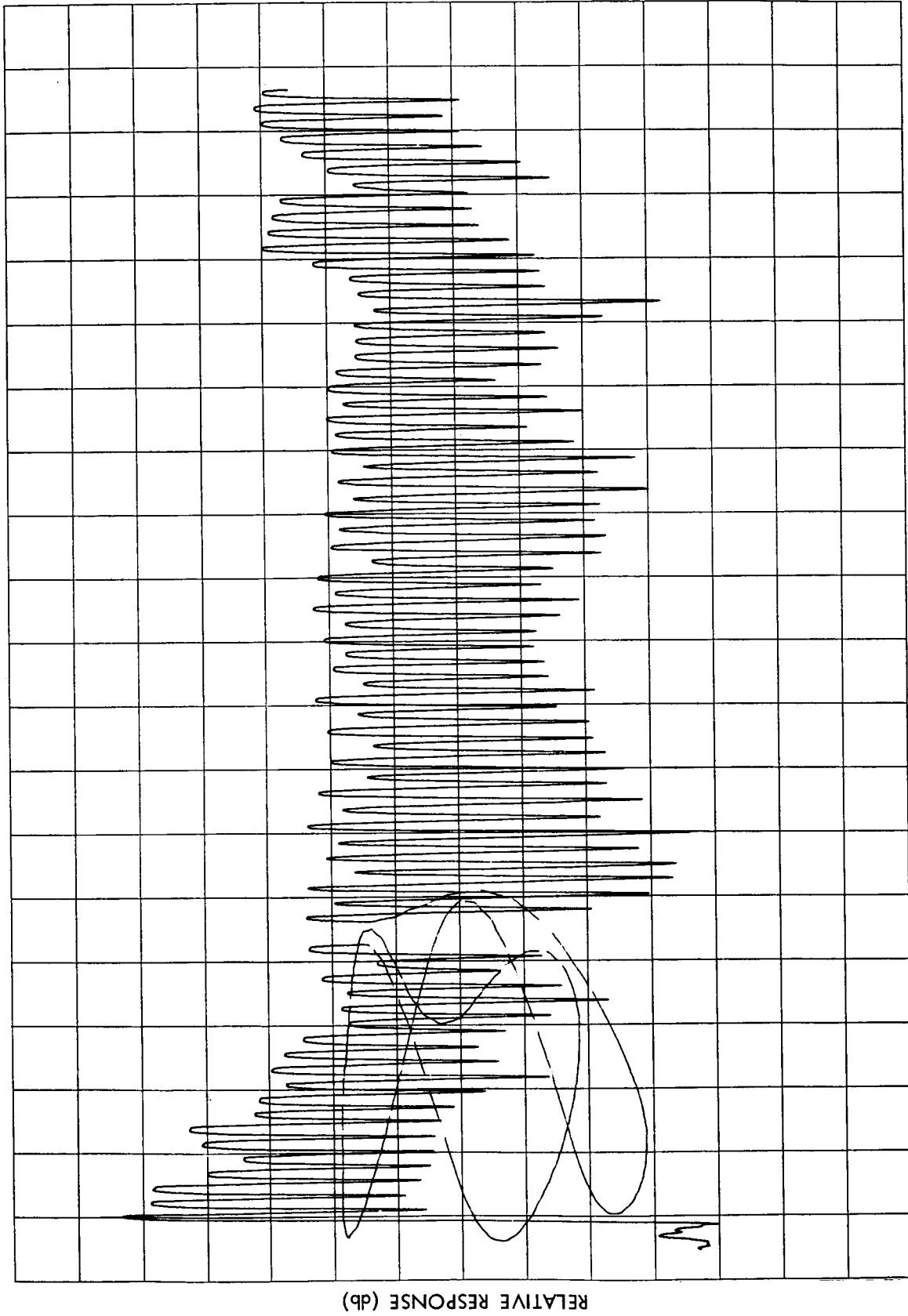


Figure 11. Plot of Output of General Radio Random Noise  
Generator Signal Fed Through MB Electronics 80 Narrow  
Band-Pass Filter Analyzer  
Sweep Time: 200 seconds and 80 seconds



FREQUENCY (LINEAR)

Figure 12. P-21 Exciter Response for Random Test of 2 March 1961

Vibration Axis: Thrust Axis  
Accelerometer Location: Exciter Table  
Scale Factor: 10 db/inch  
Input: .03 g<sup>2</sup>/cps



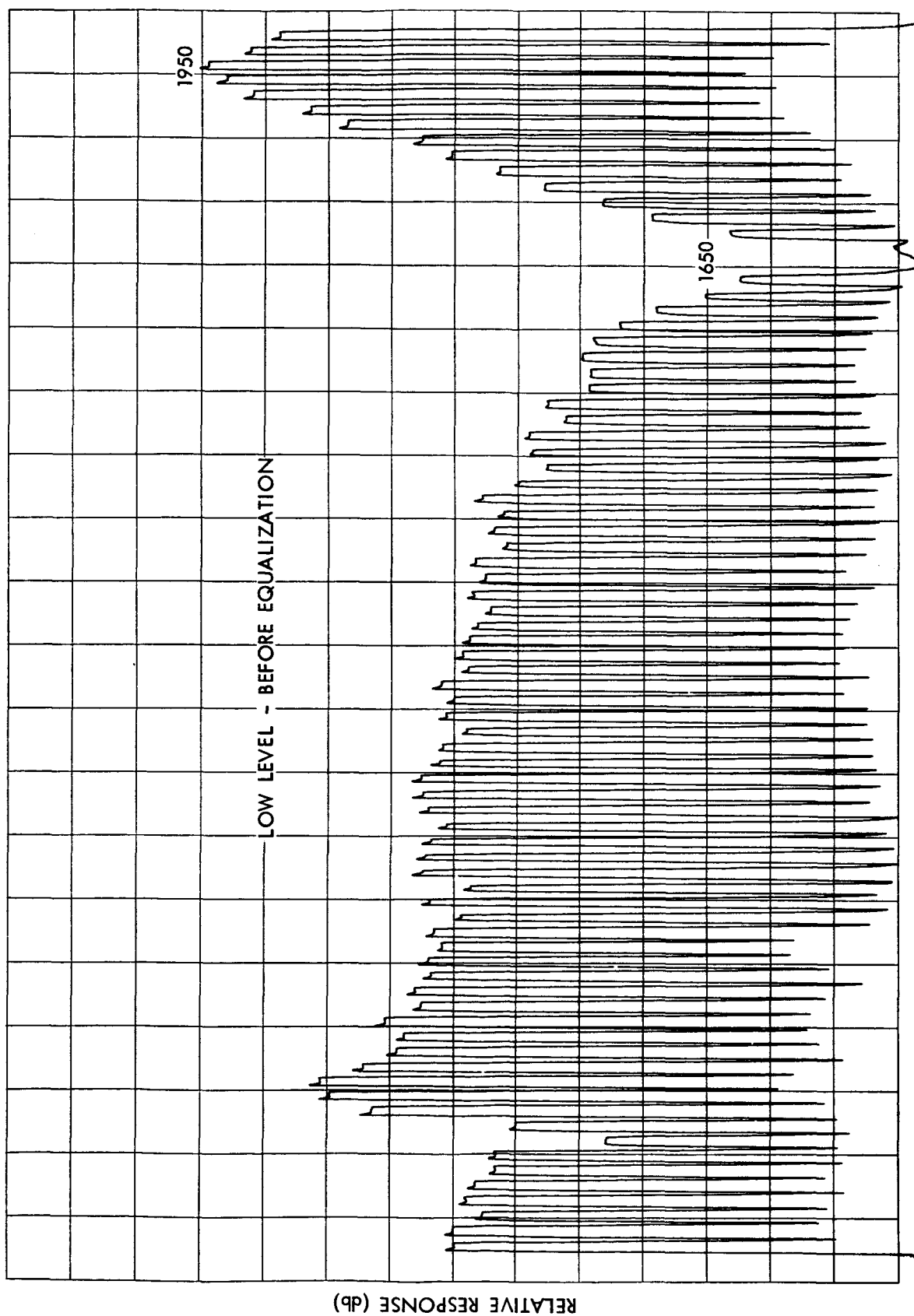


Figure 13. P-21 Exciter Response for Random Test of 2 March 1961

Vibration Axis: Thrust Axis

Accelerometer Location: Exciter Table

Scale Factor: 10 db/inch

Input: Low Level, Before Equalization

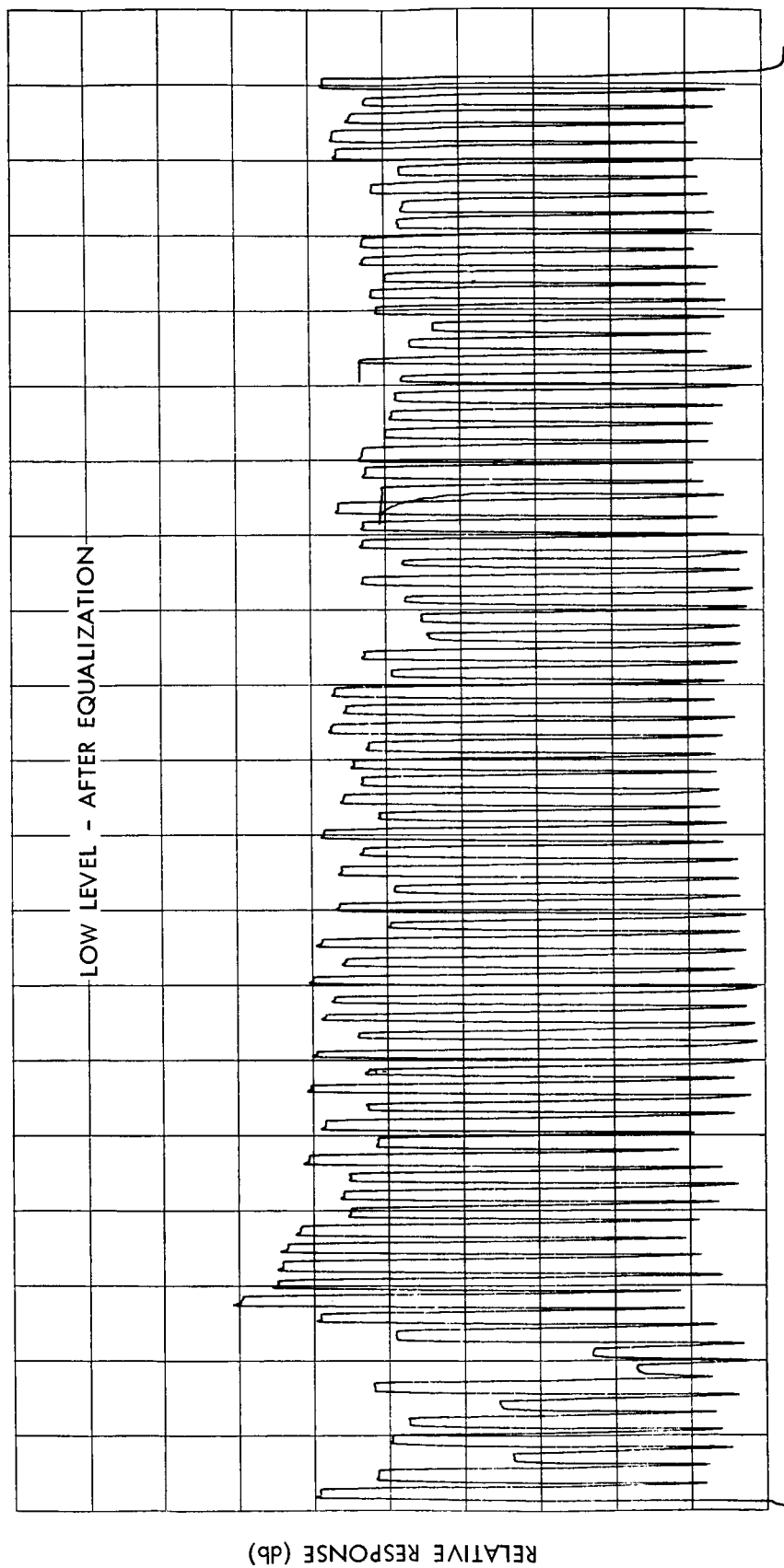


Figure 14. P-21 Exciter Response for Random Test of 2 March 1961

Vibration Axis: Thrust Axis

Accelerometer Location: Exciter Table

Input: Low Level, After Equalization

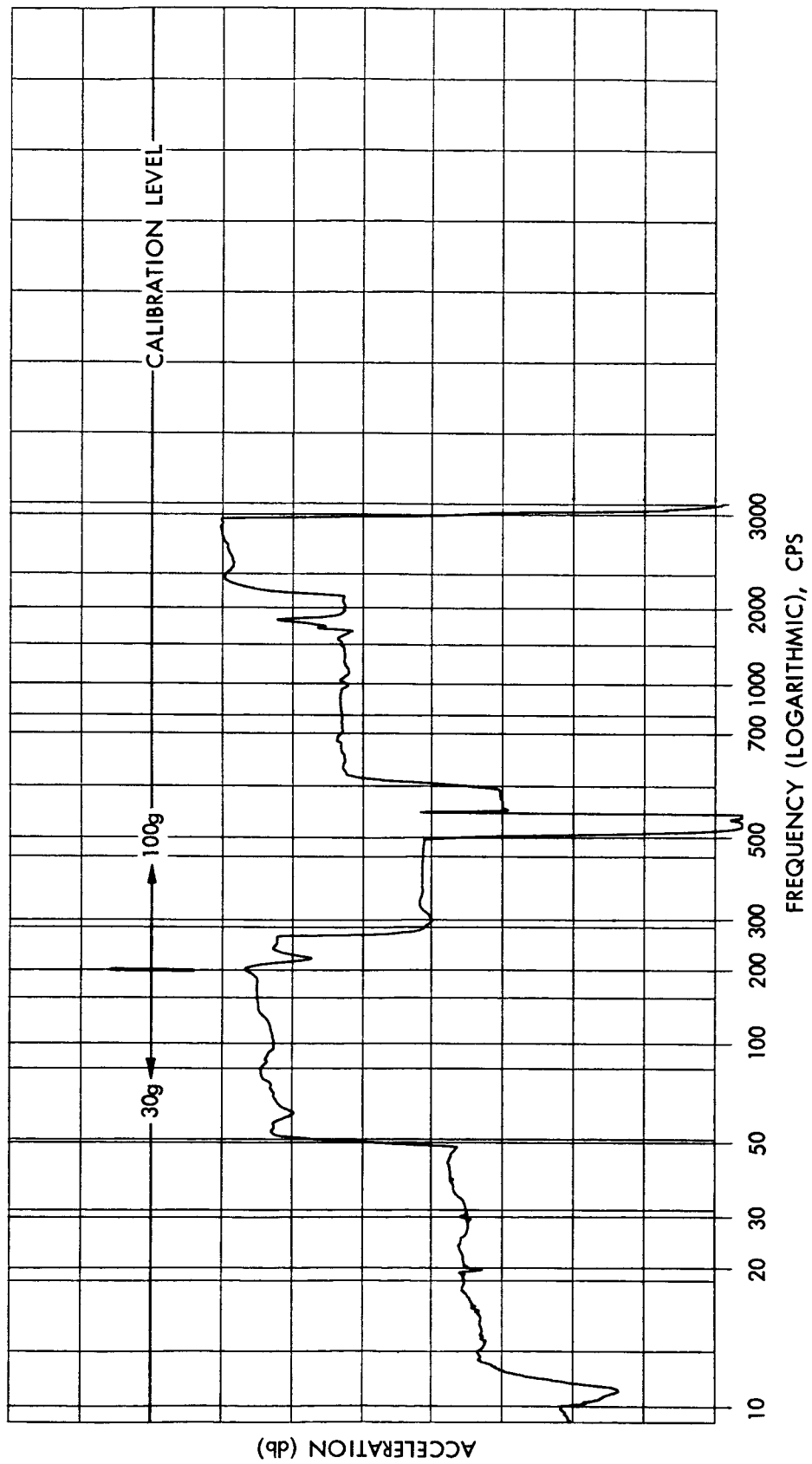


Figure 15. P-21 Sinusoidal Vibration Test of 3 March 1961

Vibration Axis: Thrust Axis

Accelerometer Location: Exciter Table

Scale Factor: 10 db/inch

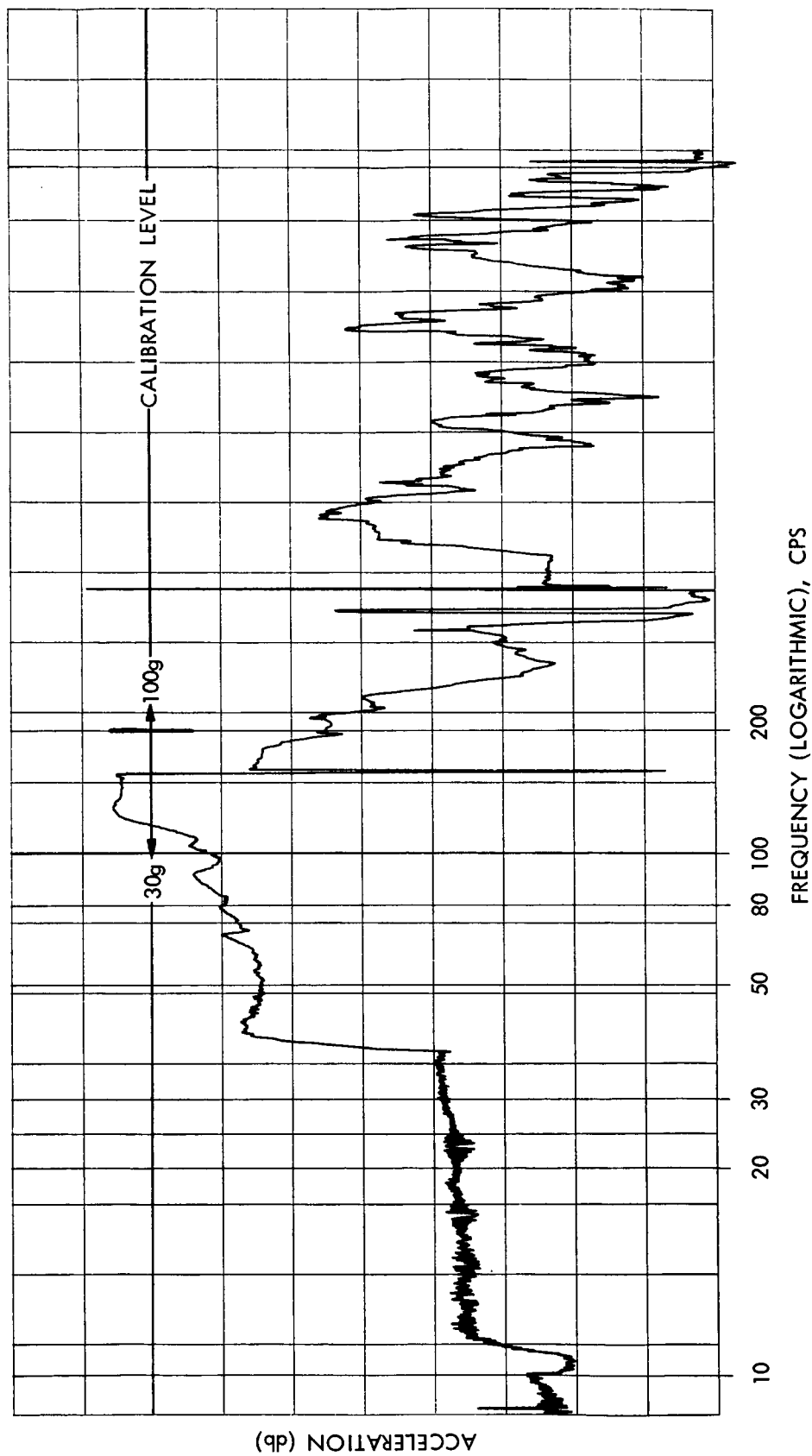


Figure 16. P-21 Sinusoidal Vibration Test of 3 March 1961  
 Vibration Axis: Thrust Axis  
 Accelerometer Location: Top of Payload Frustrum  
 Scale Factor: 10 db/inch

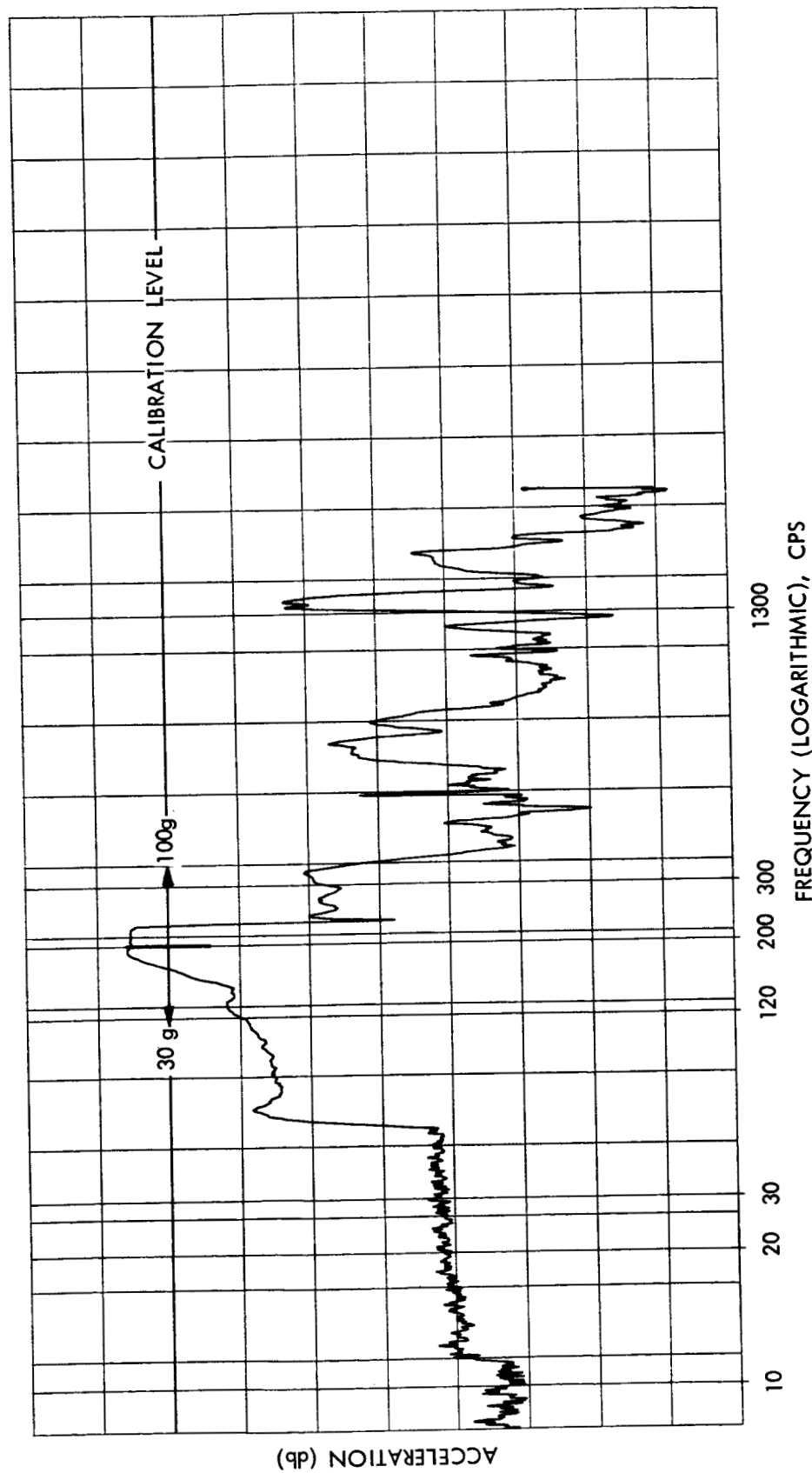


Figure 17. P-21 Sinusoidal Vibration Test of 3 March 1961

Vibration Axis: Thrust Axis

Accelerometer Location: Top of Payload Transmitter

Scale Factor: 10 db/inch

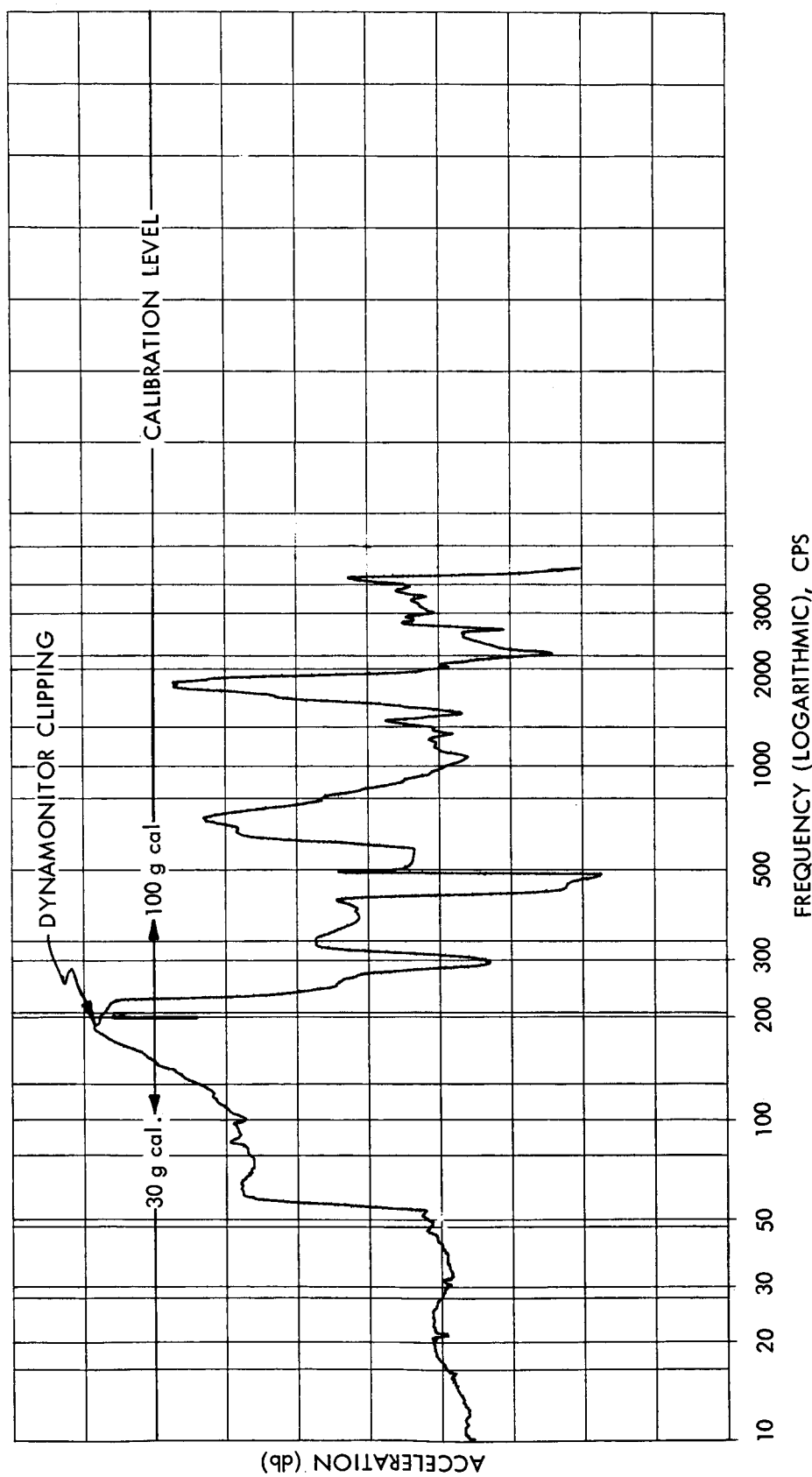


Figure 18. P-21 Sinusoidal Vibration Test of 3 March 1961  
 Vibration Axis: Thrust Axis  
 Accelerometer Location: Telemetry Base Plate  
 Scale Factor: 10 db/inch

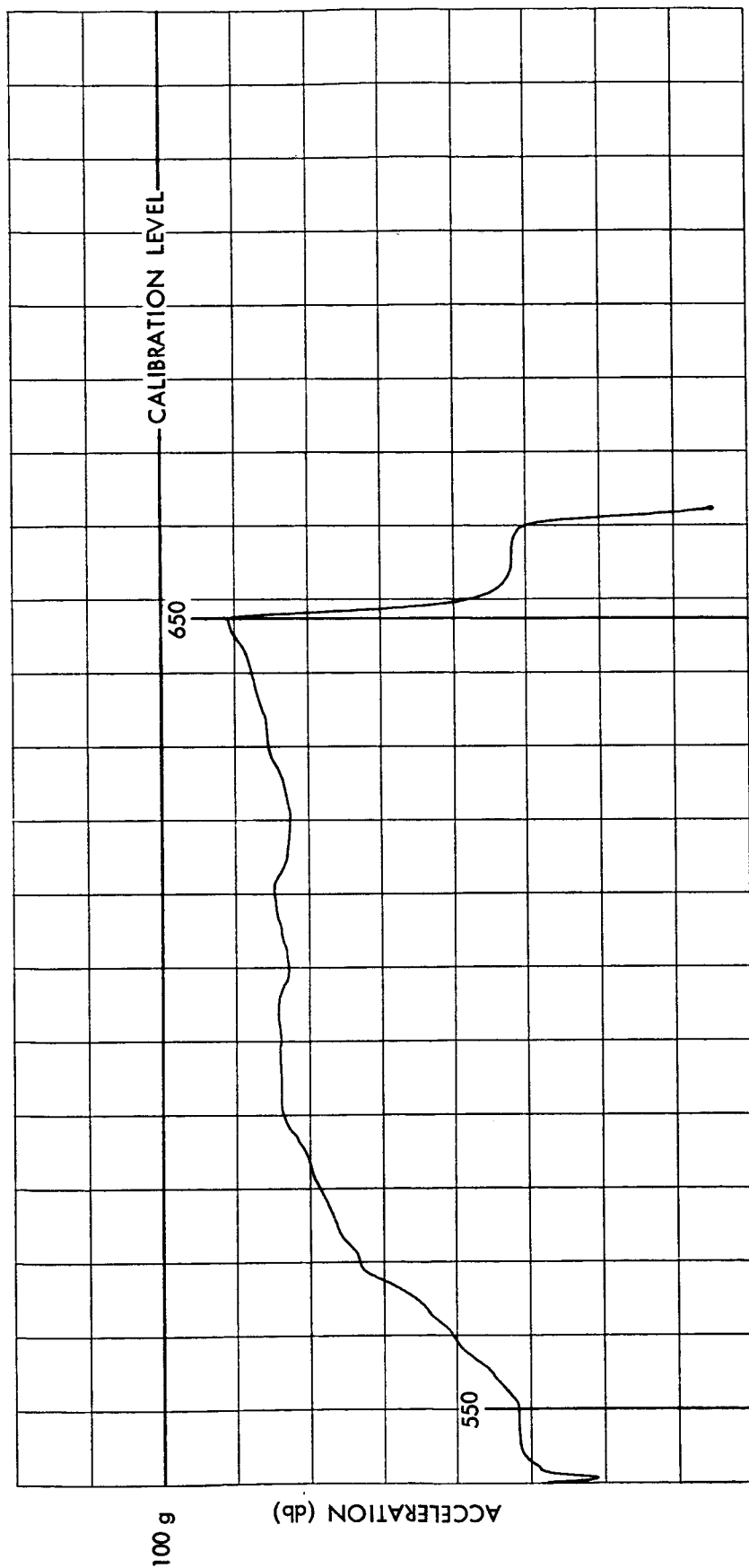
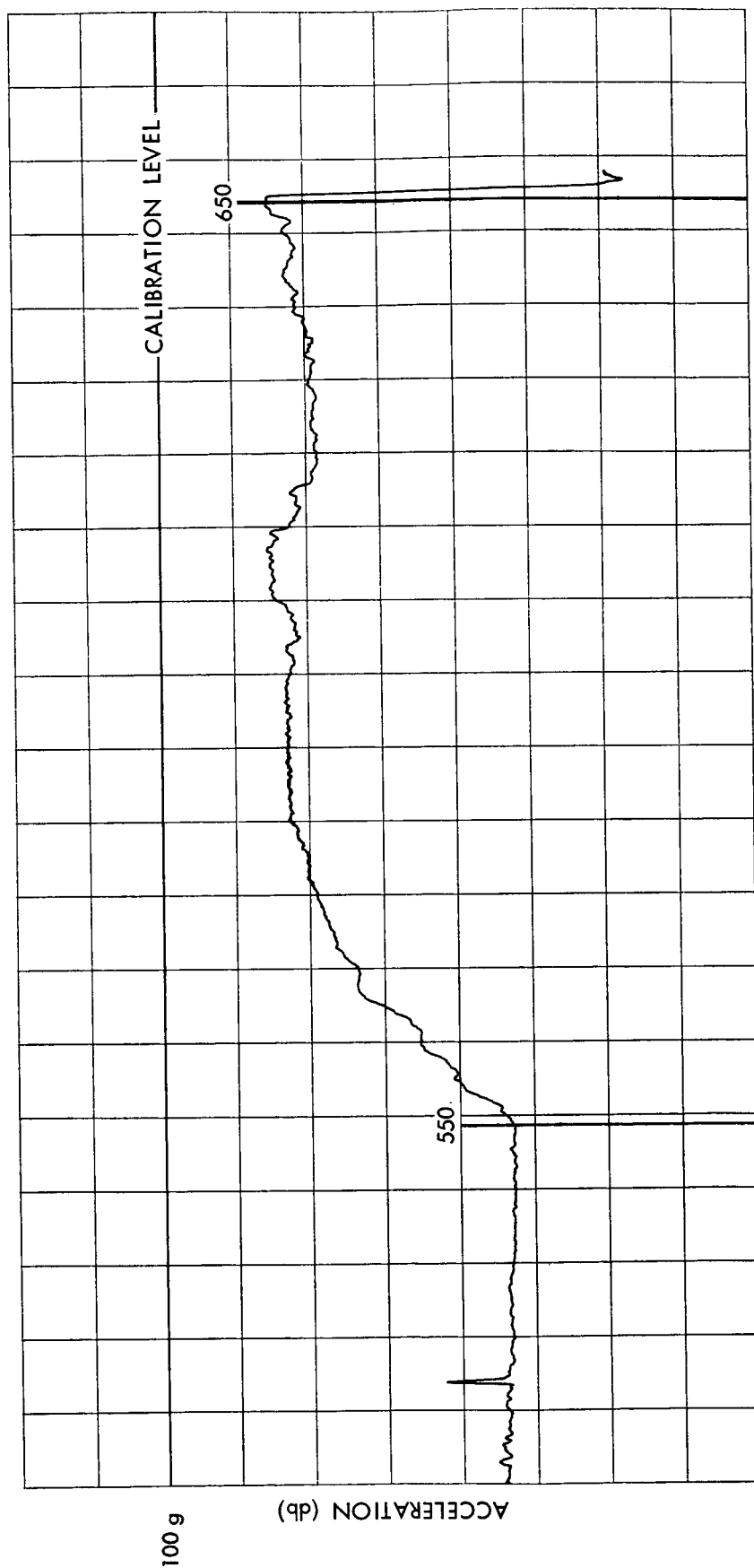


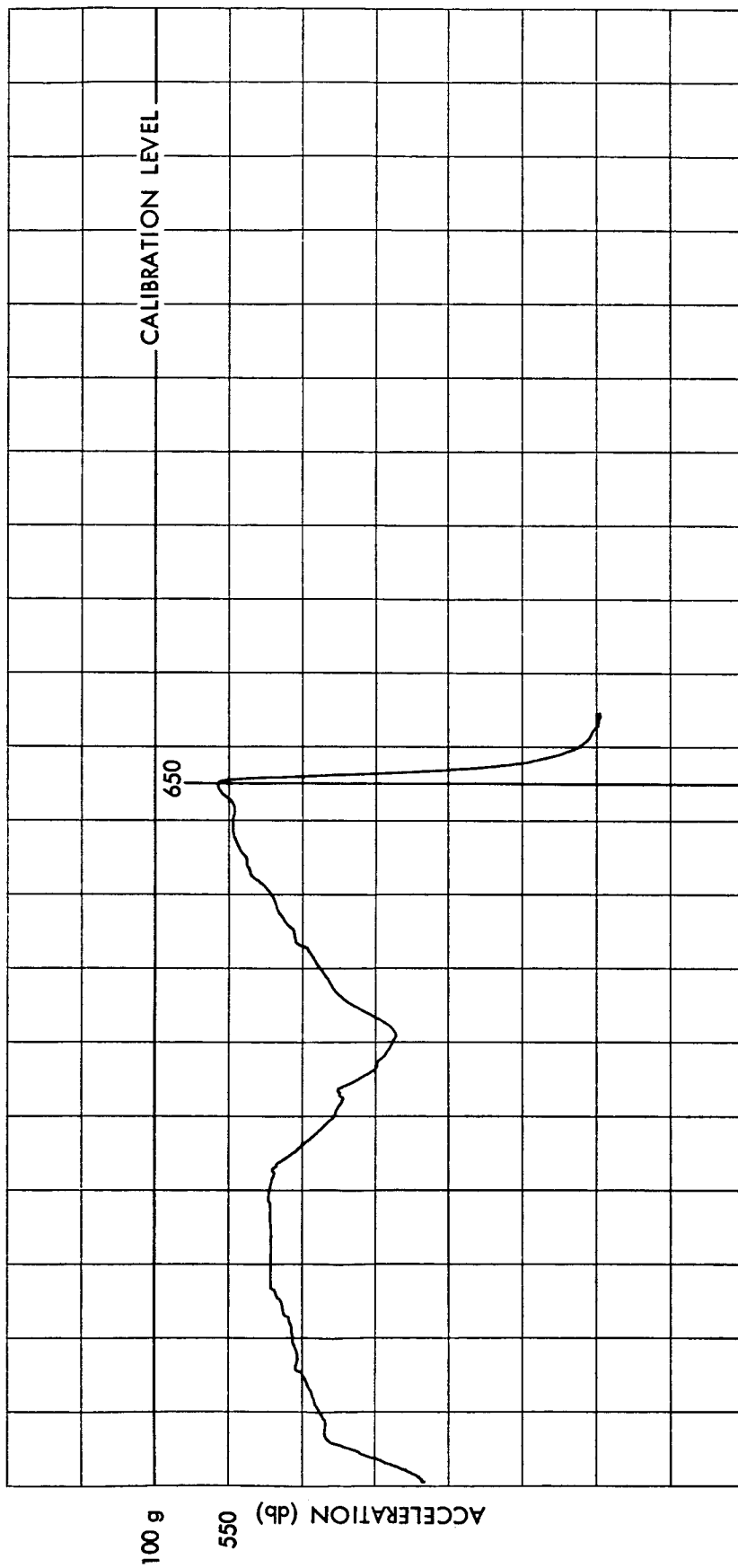
Figure 19. P-21 600 cps Test of 3 March 1961  
Vibration Axis: Thrust Axis  
Accelerometer Location: Top of Force Fixture  
Scale Factor: 10 db/inch



FREQUENCY (LOGARITHMIC), CPS

Figure 20. P-21 600 cps Test of 3 March 1961  
Vibration Axis: Thrust Axis  
Accelerometer Location: Top of Payload Frustrum  
Scale Factor: 10 db/inch





FREQUENCY (LOGARITHMIC), CPS  
 Figure 21. 600 cps Test of 3 March 1961  
 Vibration Axis: Thrust Axis  
 Accelerometer Location: Top of Transmitter

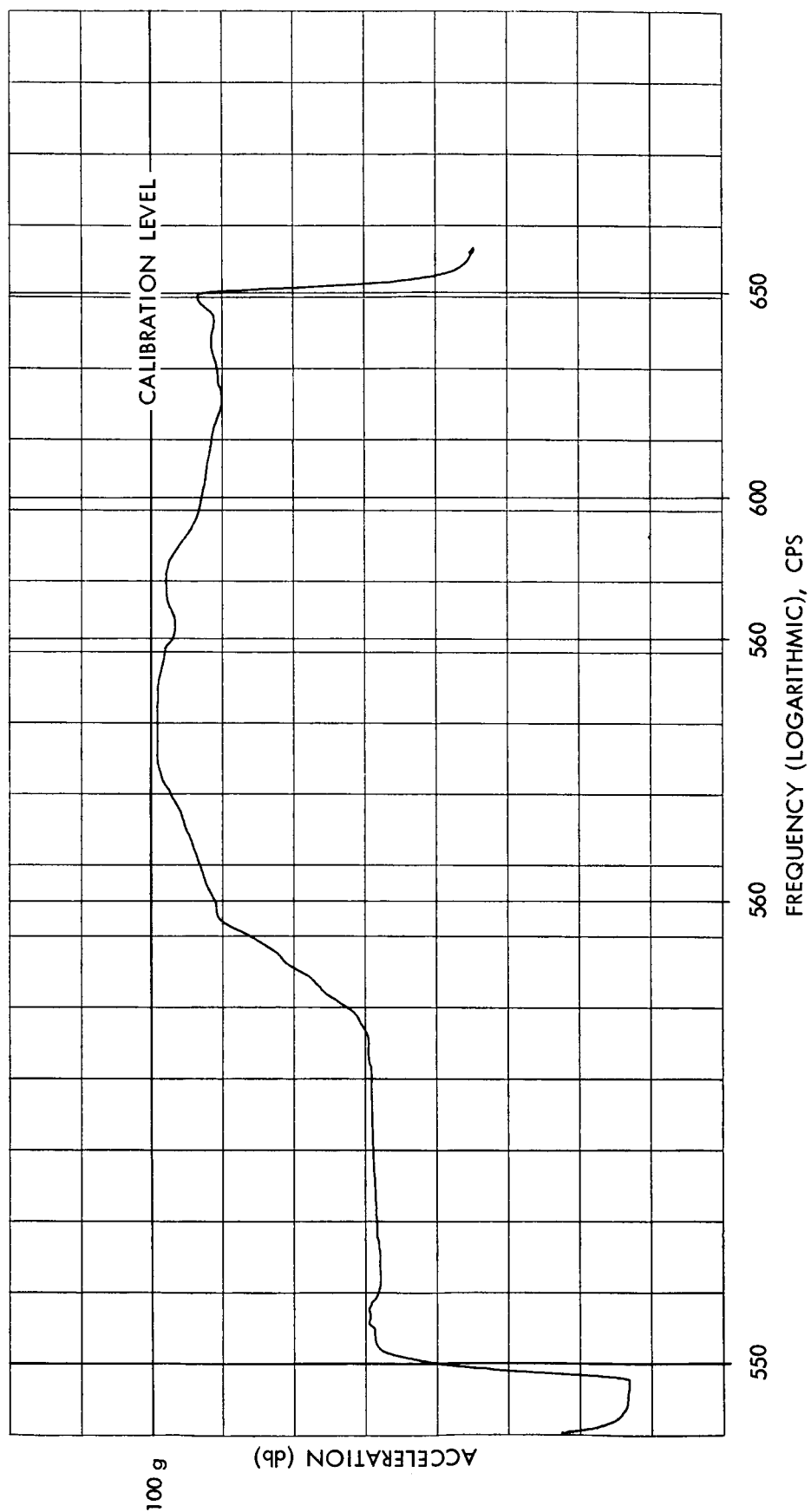


Figure 22. P-21 600 cps Vibration Test of 3 March 1961

Vibration Axis: Thrust Axis

Accelerometer Location: Telemetry Base Plate

Scale Factor: 10 db/inch

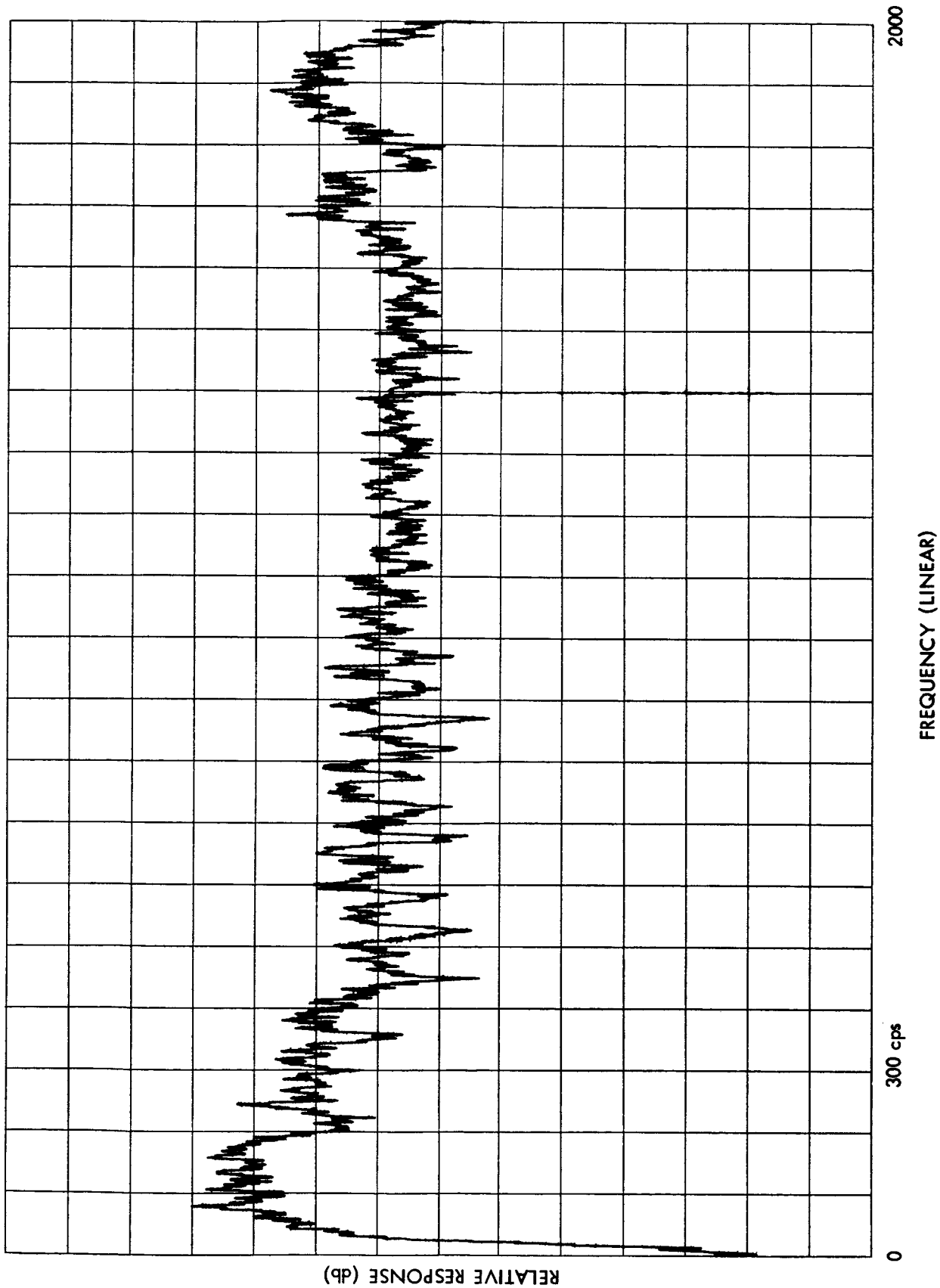
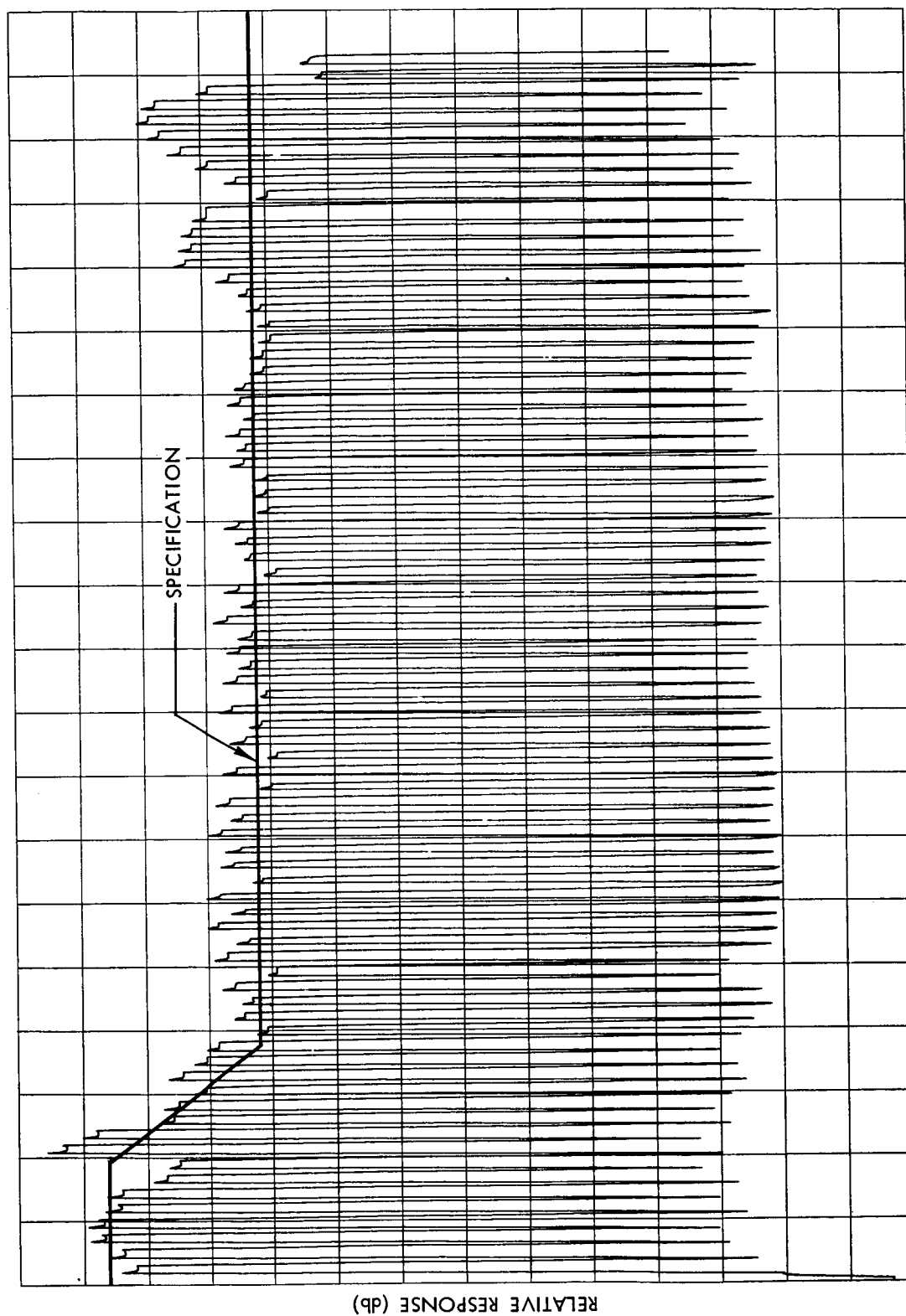


Figure 23. P-21 Random Vibration Test of 2 March 1961  
Vibration Axis: Thrust Axis  
Accelerometer Location: Exciter Table  
Scale Factor: 10 db/inch



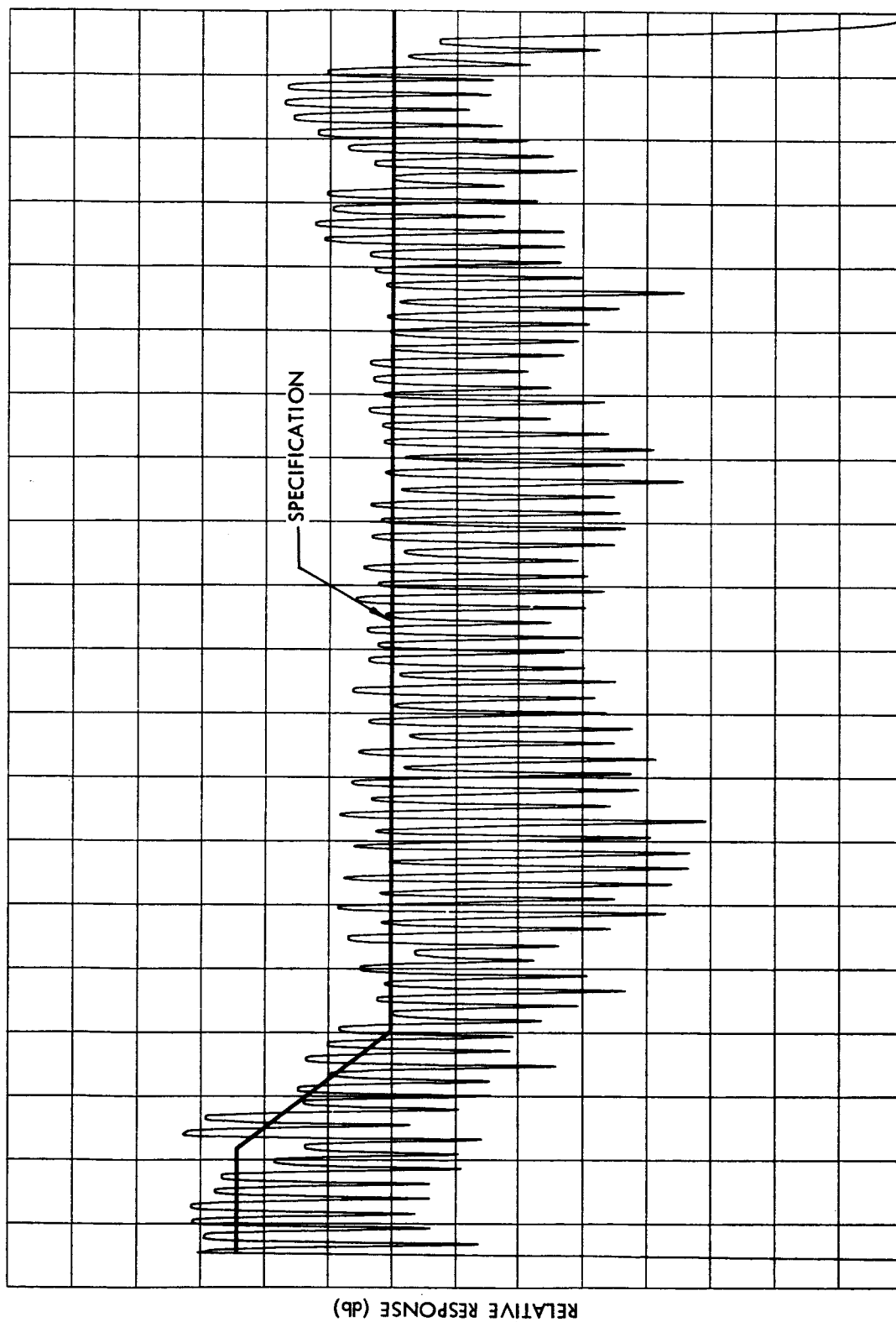
FREQUENCY (LINEAR)

Figure 24. P-21 Random Vibration Test of 2 March 1961

Vibration Axis: Thrust Axis

Accelerometer Location: Exciter Table

Scale Factor: 10 db/inch



FREQUENCY (LINEAR)

Figure 25. P-21 Random Vibration Test of 2 March 1961

Vibration Axis: Thrust Axis

Accelerometer Location: Exciter Table

Scale Factor: 10 db/inch

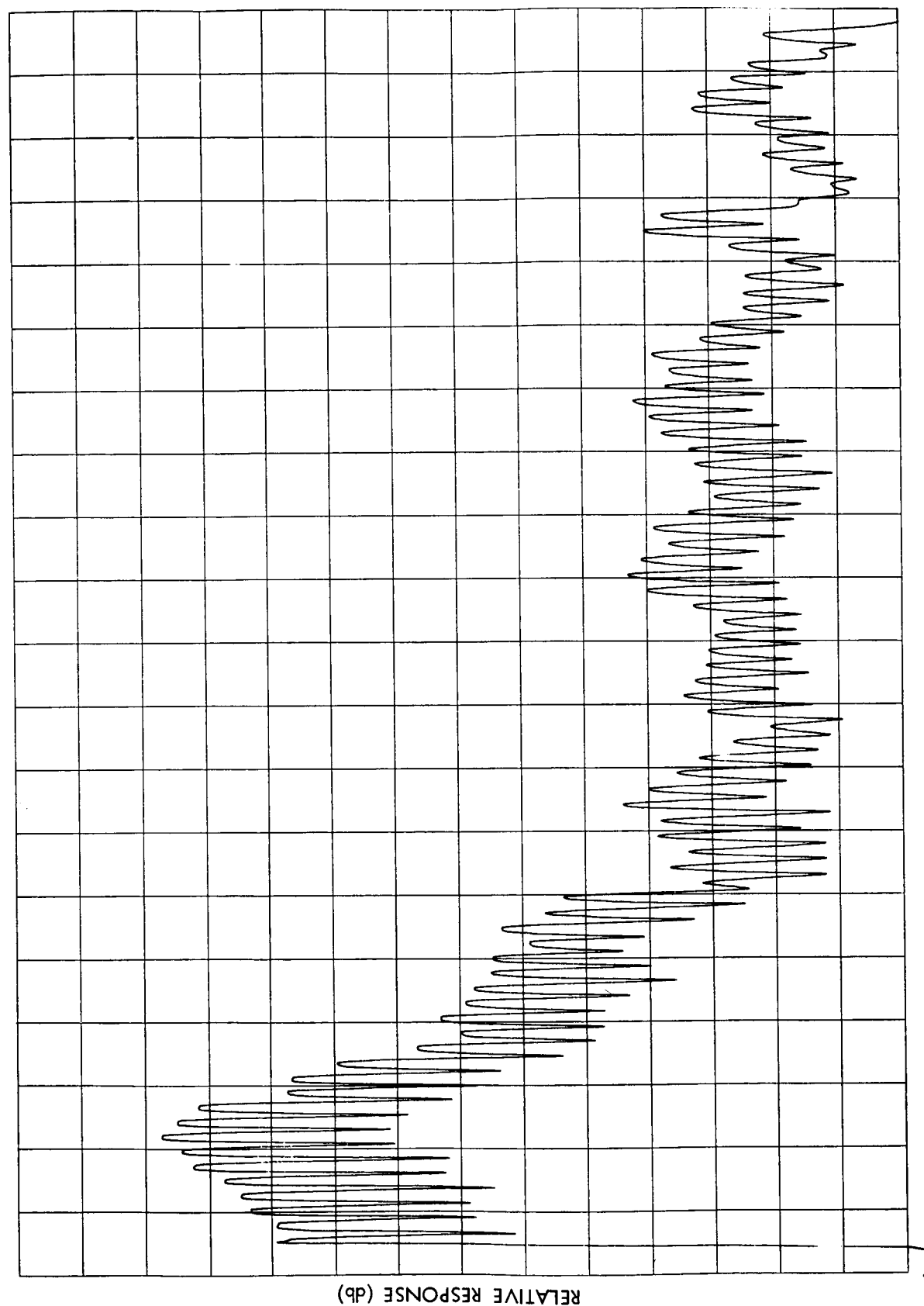


Figure 26. P-21 Random Vibration Test of 2 March 1961

Vibration Axis: Thrust Axis

Accelerometer Location: Top of Payload Frustrum

Scale Factor: 10 db/inch

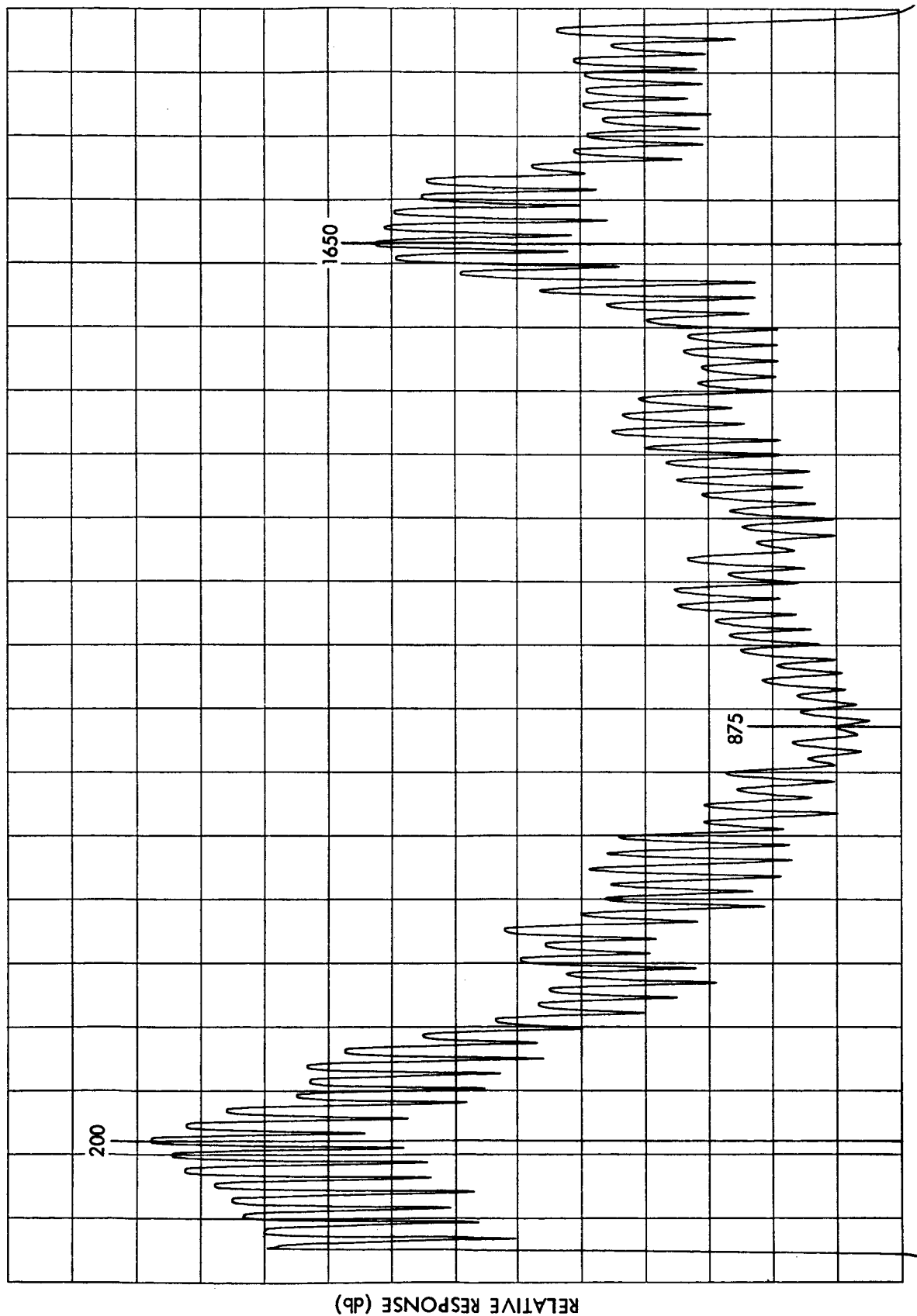


Figure 27. P-21 Random Vibration Test of 2 March 1961  
Vibration Axis: Thrust Axis  
Accelerometer Location: Top of Payload Transmitter  
Scale Factor: 10 db/inch

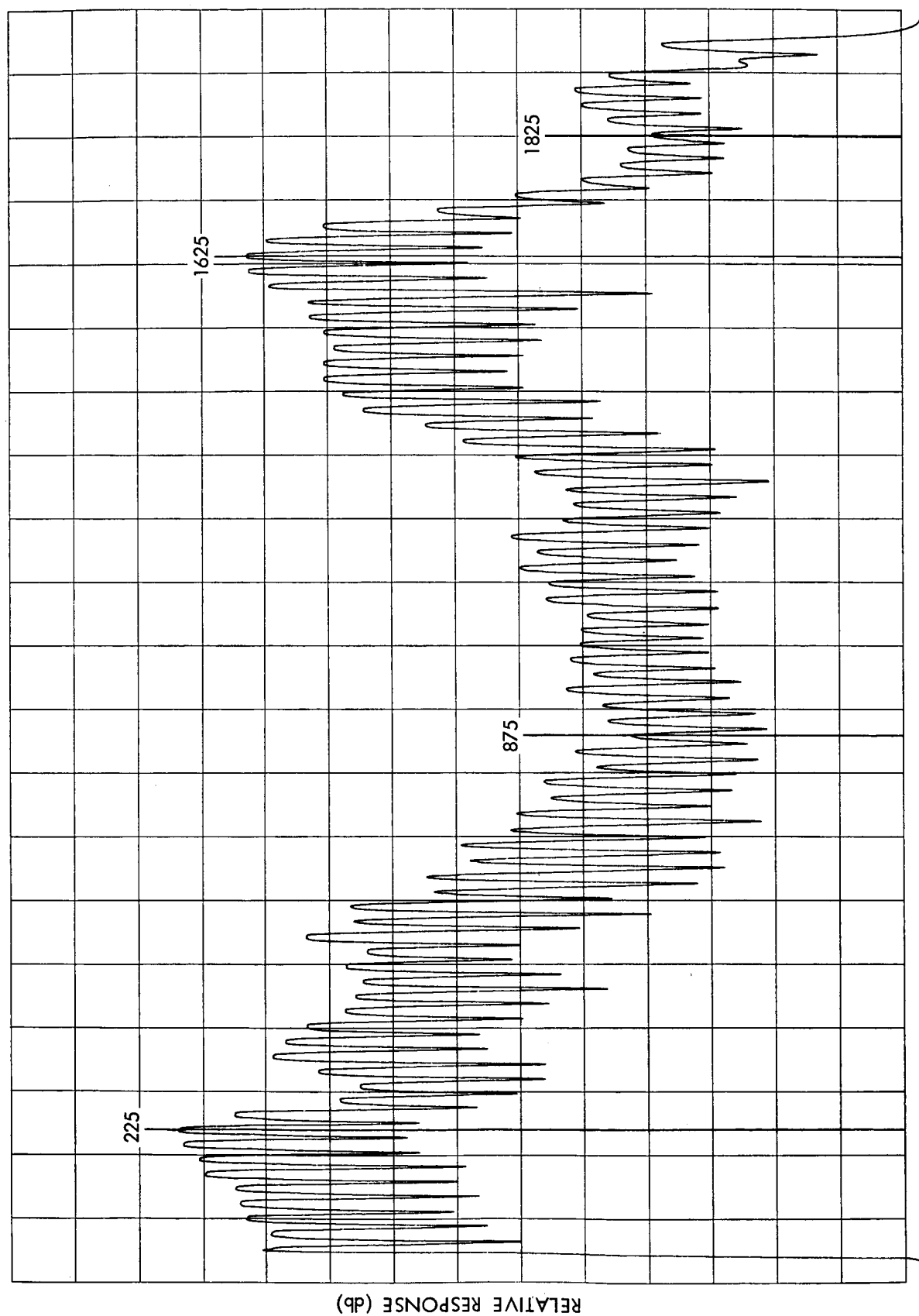
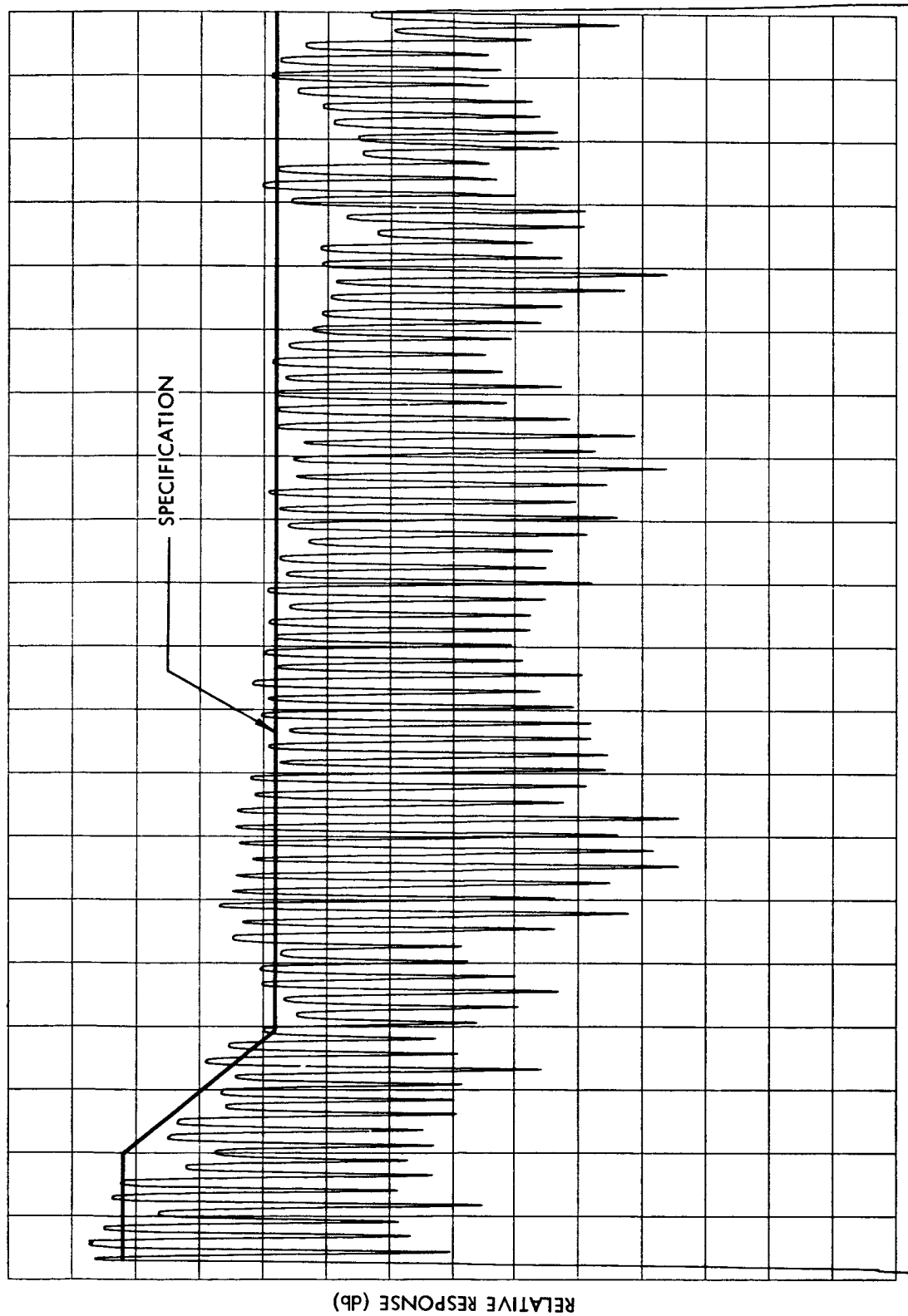


Figure 28. P-21 Random Vibration Test of 2 March 1961  
Vibration Axis: Thrust Axis  
Accelerometer Location: Telemetry Base Plate  
Scale Factor: 10 db/inch





### FREQUENCY (LINEAR)

Figure 29. P-21 Random Vibration Test of 3 March 1961

Vibration Axis: Thrust Axis

Accelerometer Location: Exciter Table

Scale Factor: 10 db/inch

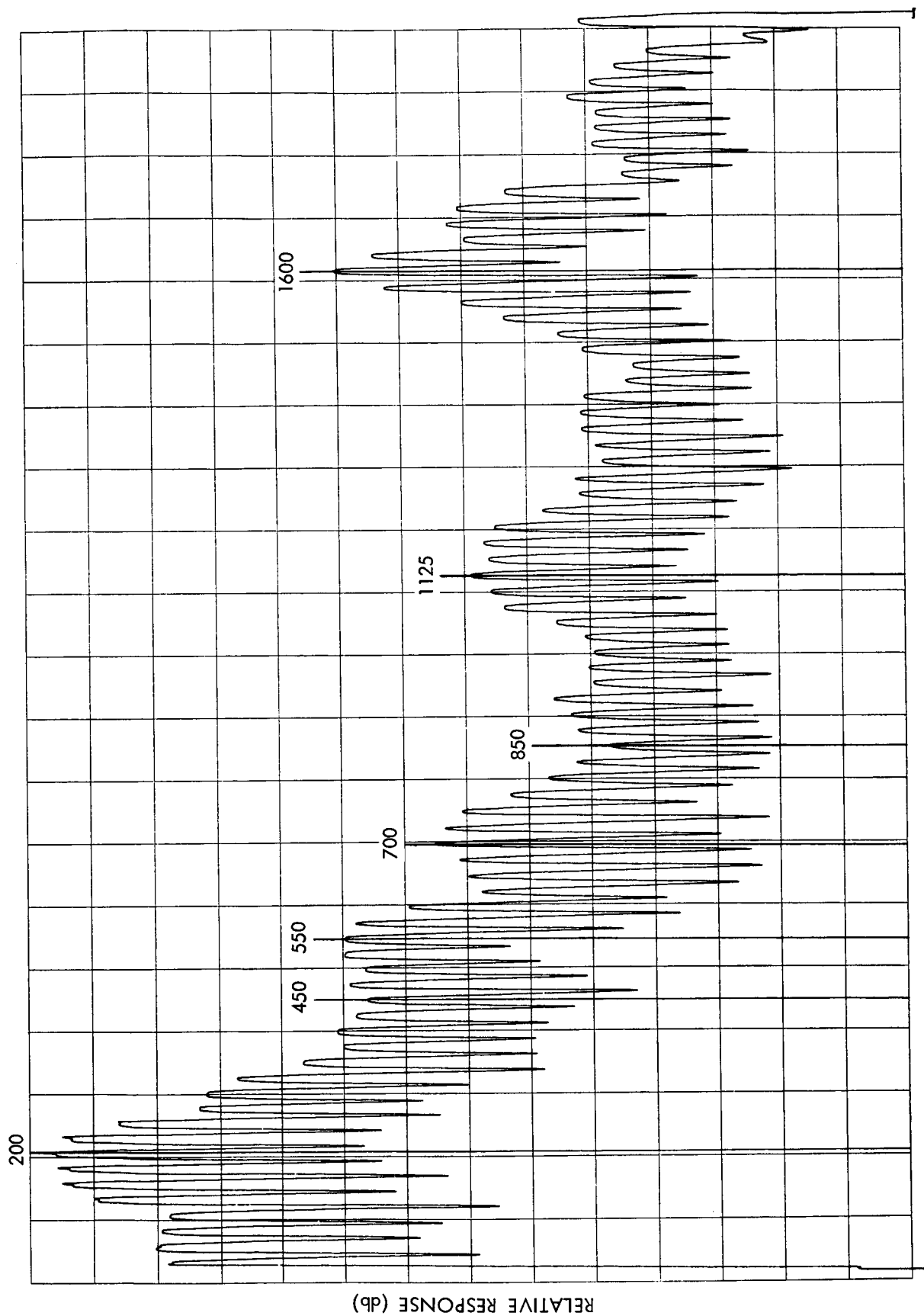
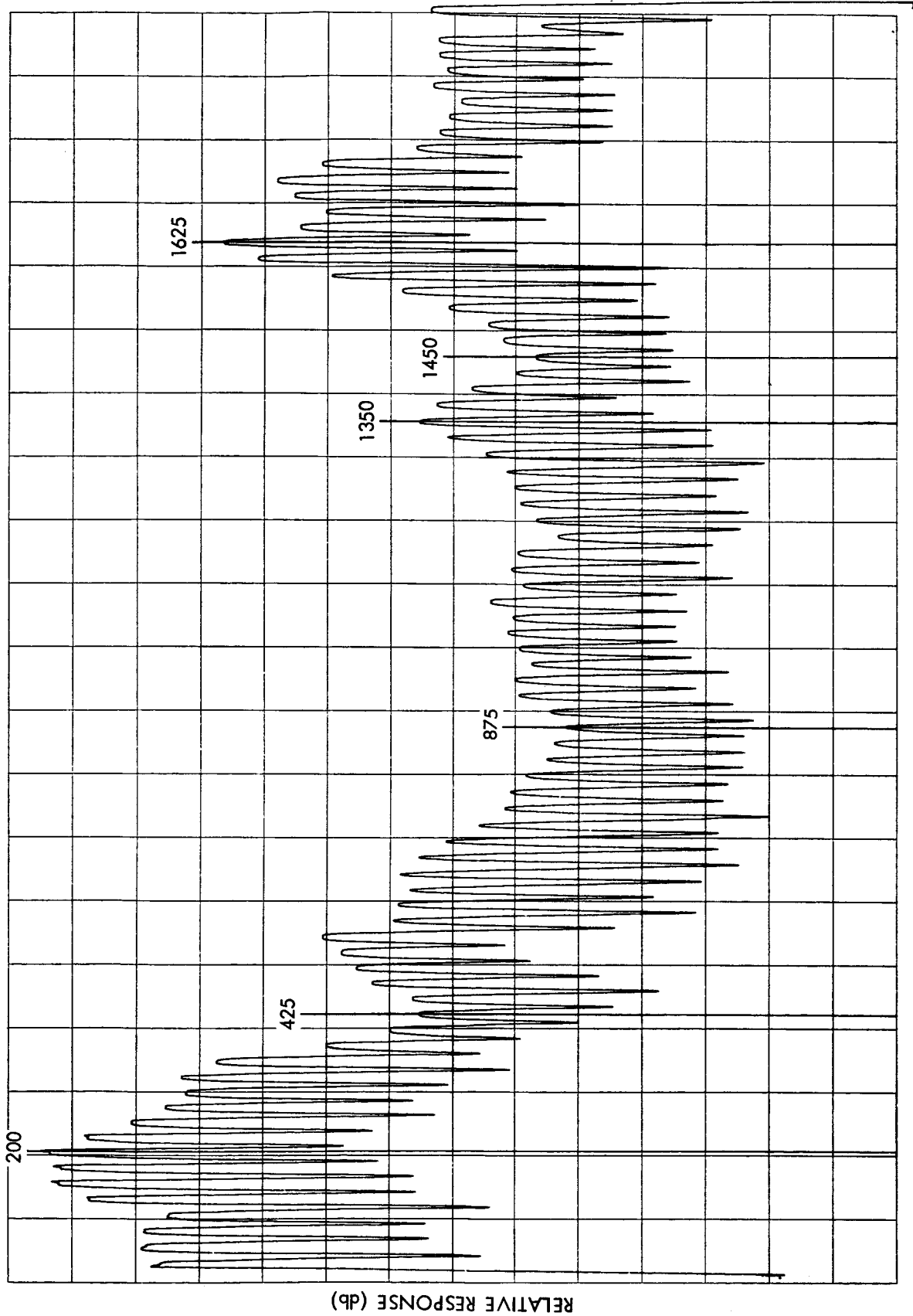


Figure 30. P-21 Random Vibration Test of 3 March 1961  
Vibration Axis: Thrust Axis  
Accelerometer Location: Top of Payload Frustrum



FREQUENCY (LINEAR)

Figure 31. P-21 Random Vibration Test of 3 March 1961  
 Vibration Axis: Thrust Axis  
 Accelerometer Location: Top of Payload Transmitter  
 Scale Factor: 10 db/inch

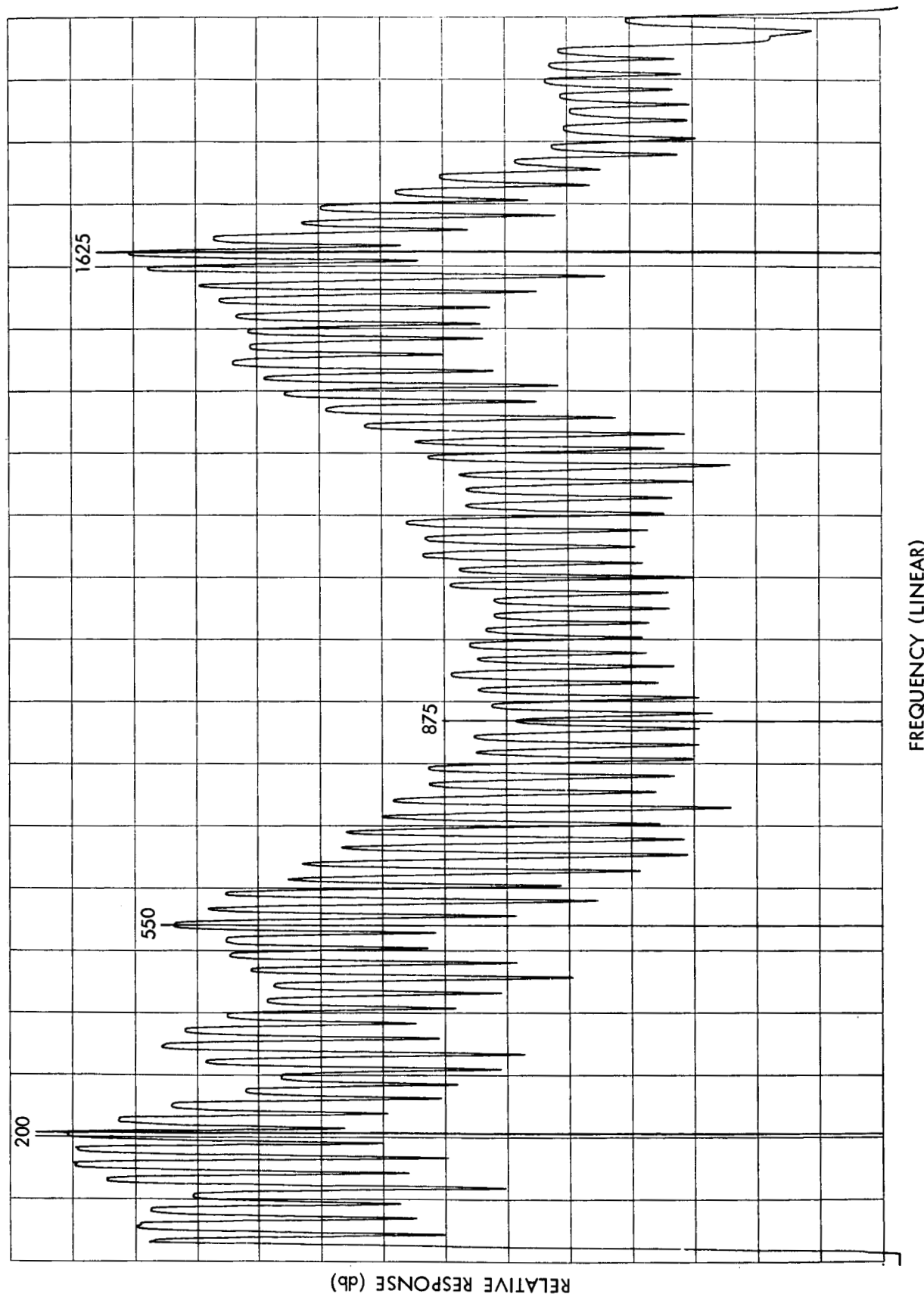


Figure 32. P-21 Random Vibration Test of 3 March 1961  
 Vibration Axis: Thrust Axis  
 Accelerometer Location: Telemetry Base Plate  
 Scale Factor: 10 db/inch

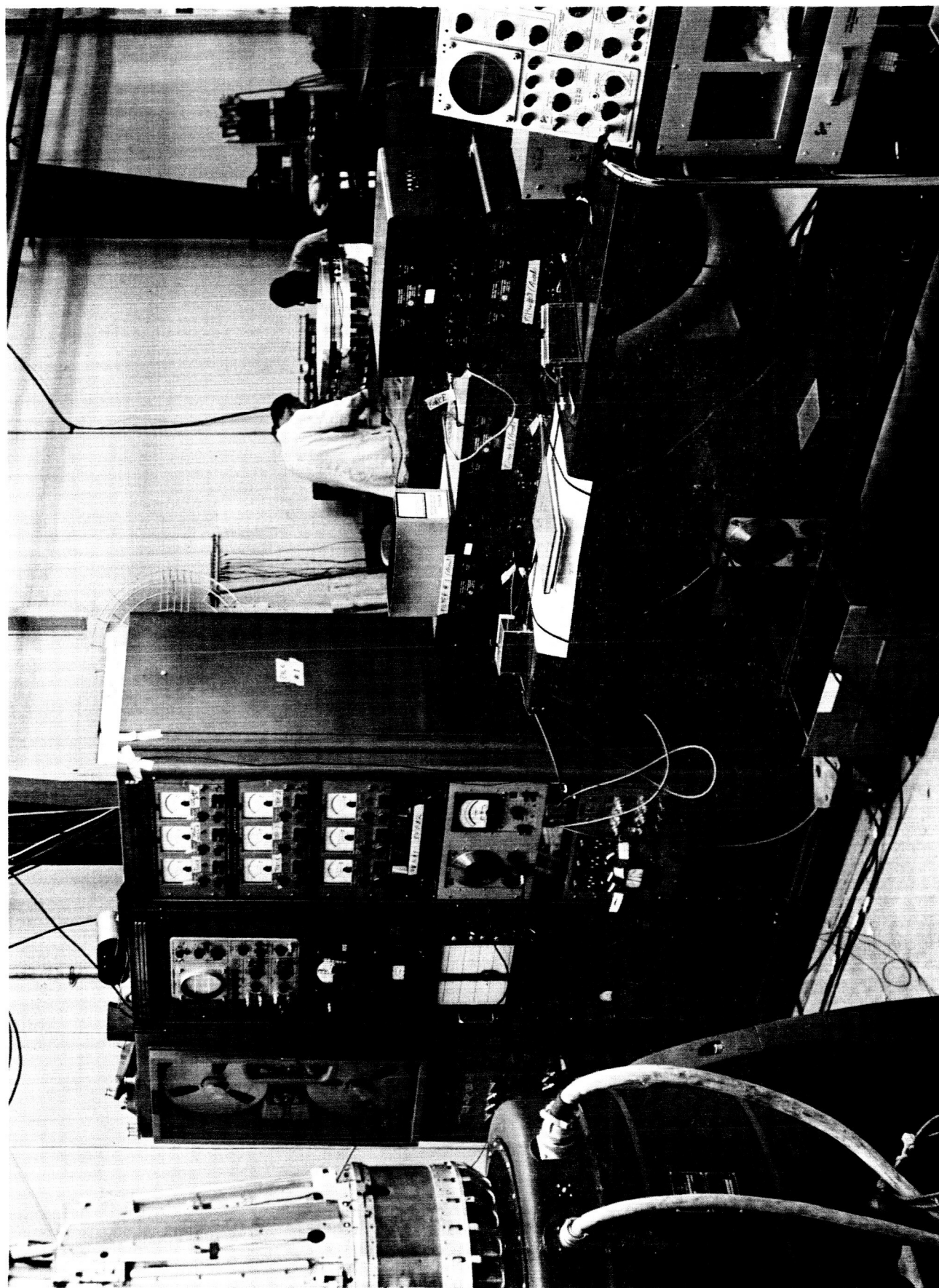


Figure 33. Data Reduction and Force Programming Equipment

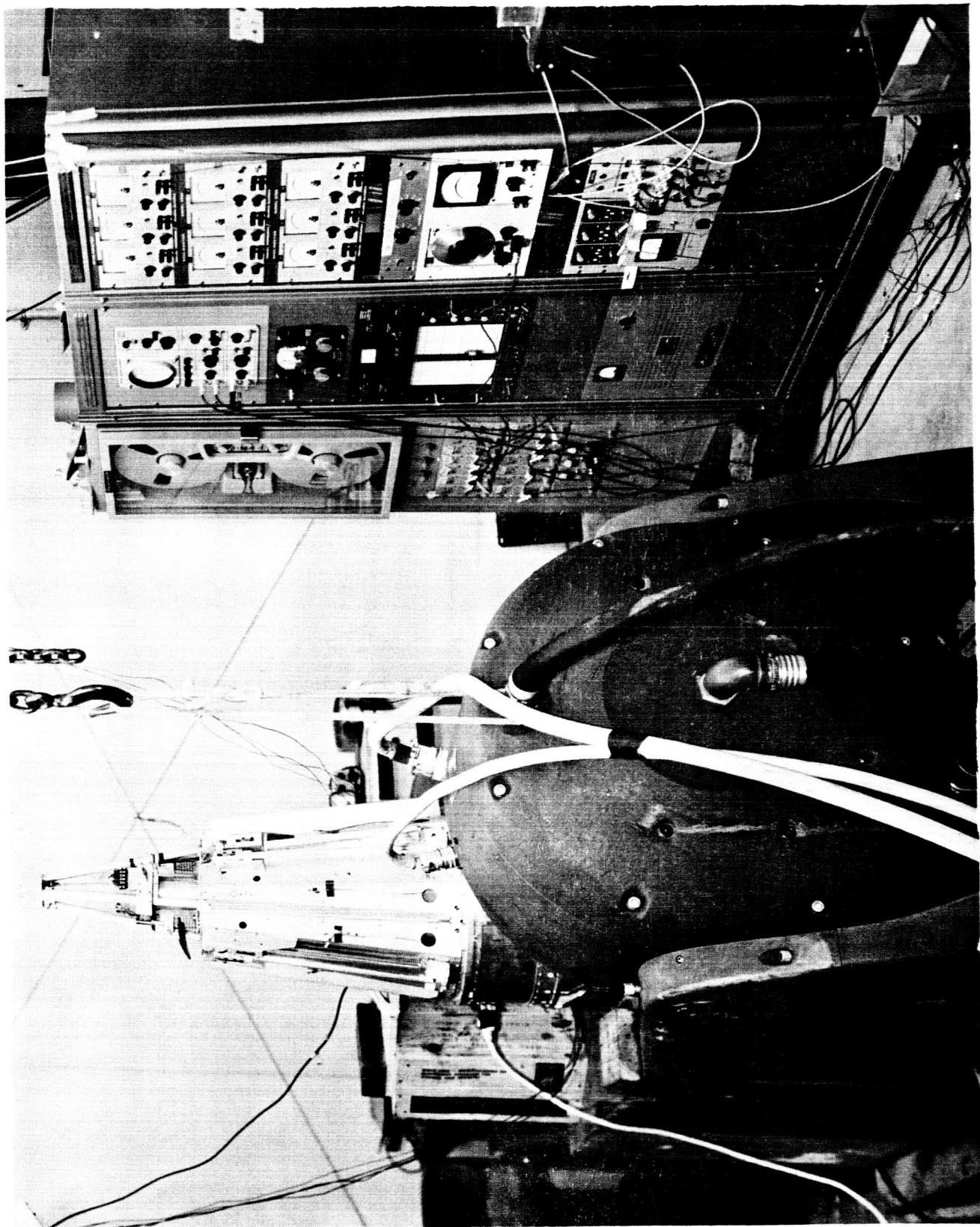


Figure 34. Vibration Exciter and Data Reduction Equipment



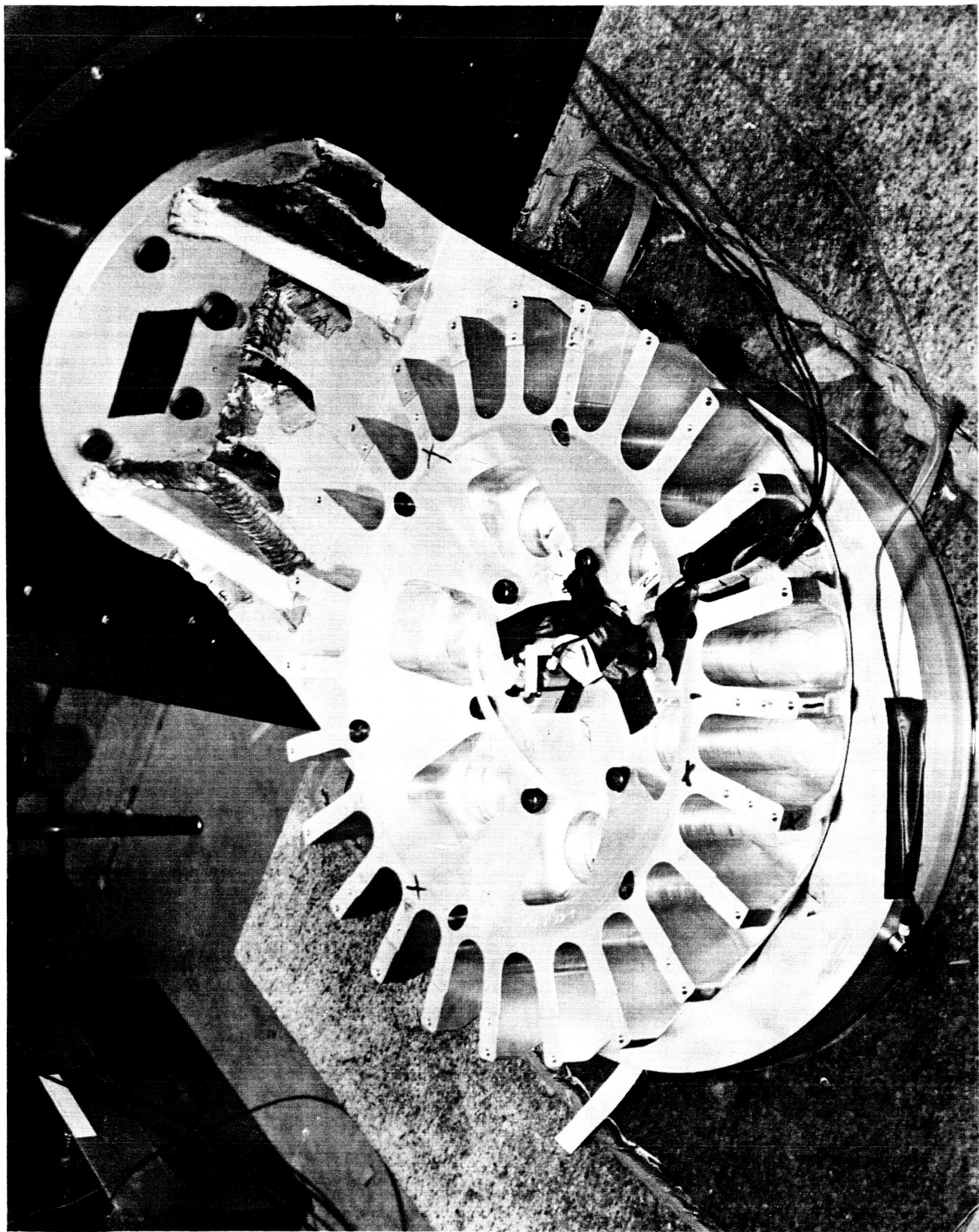


Figure 35. Horizontal Vibration Facility

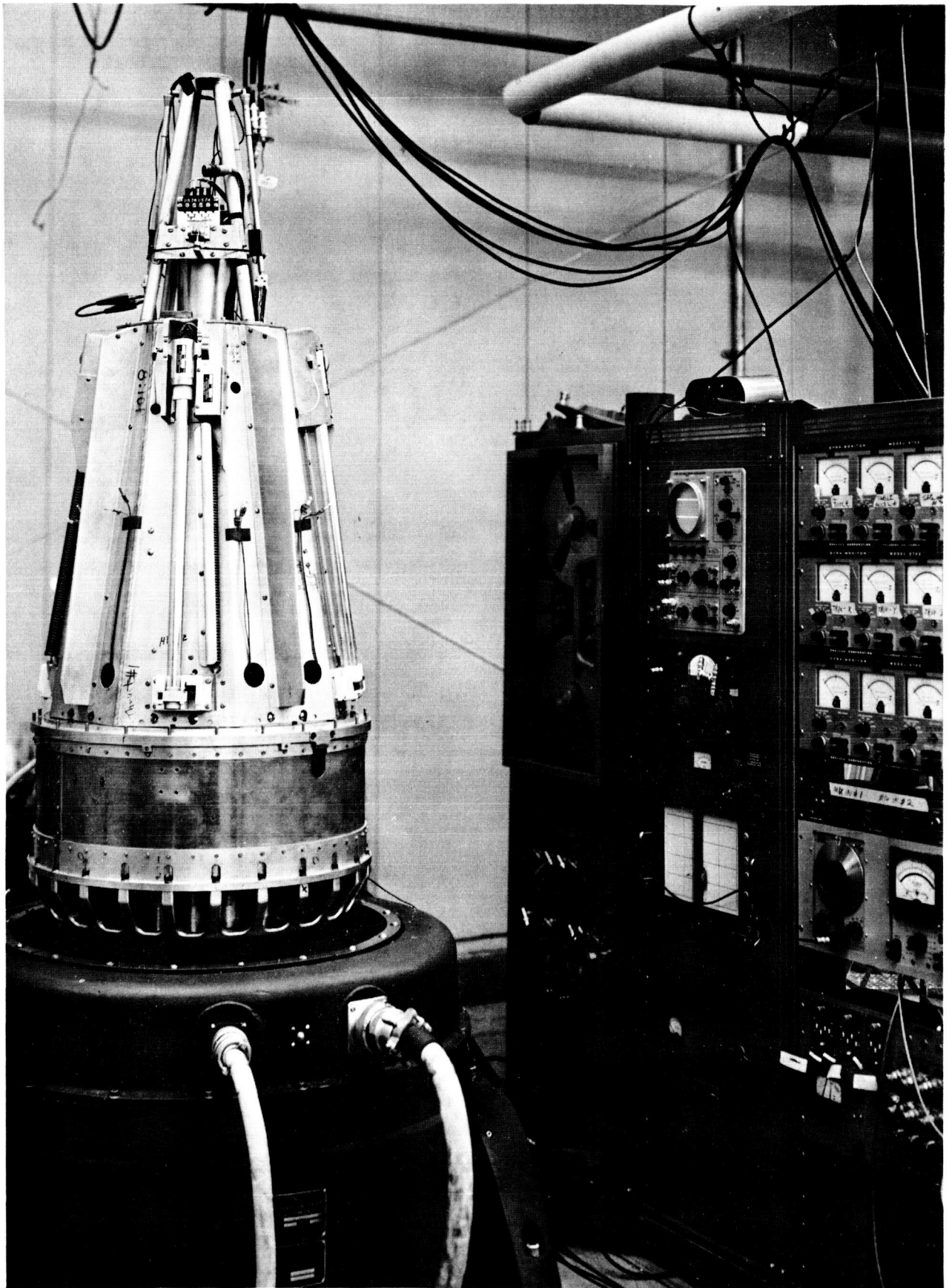


Figure 36. Vibration Exciter - Thrust Axis



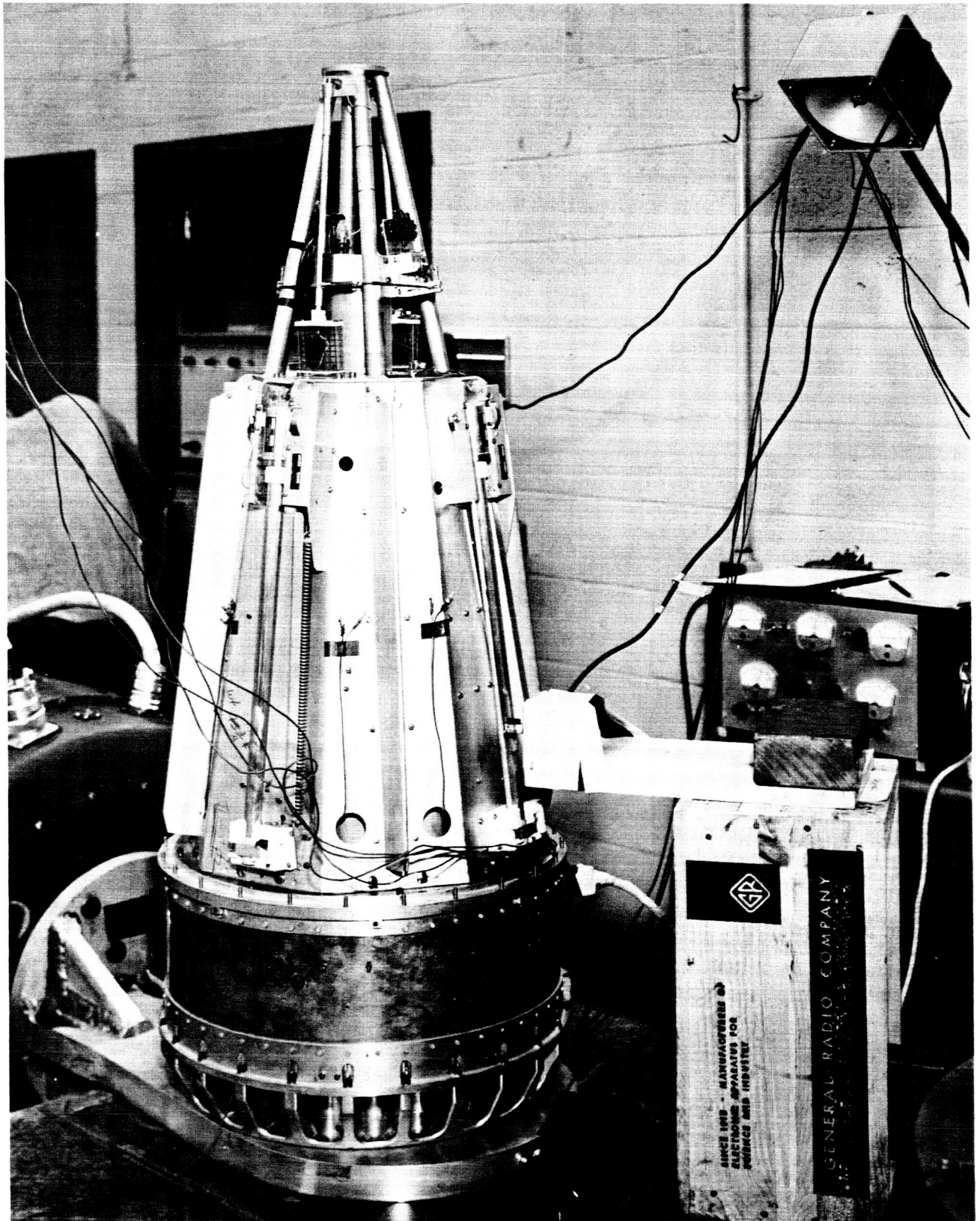


Figure 37. Horizontal Vibration Setup

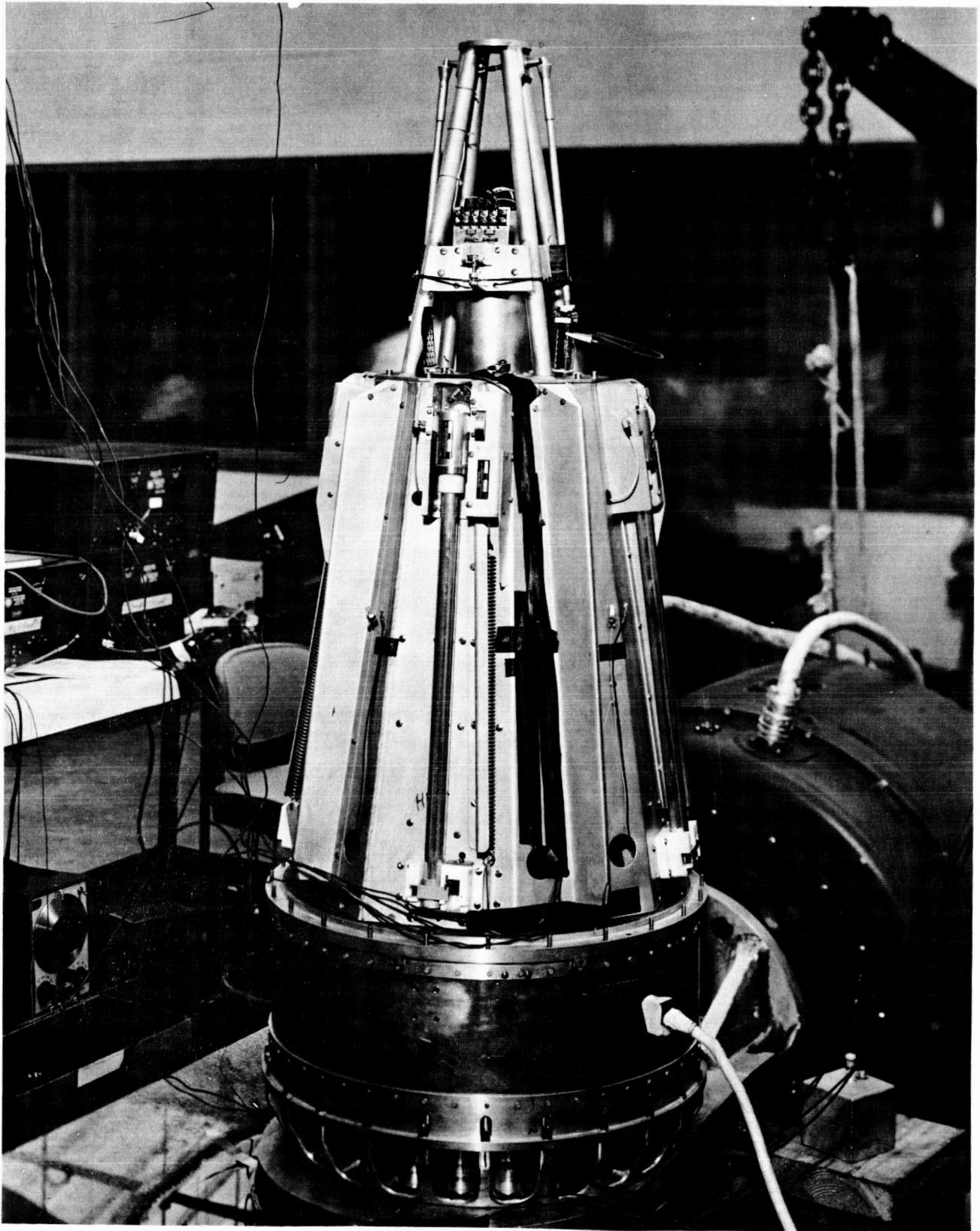
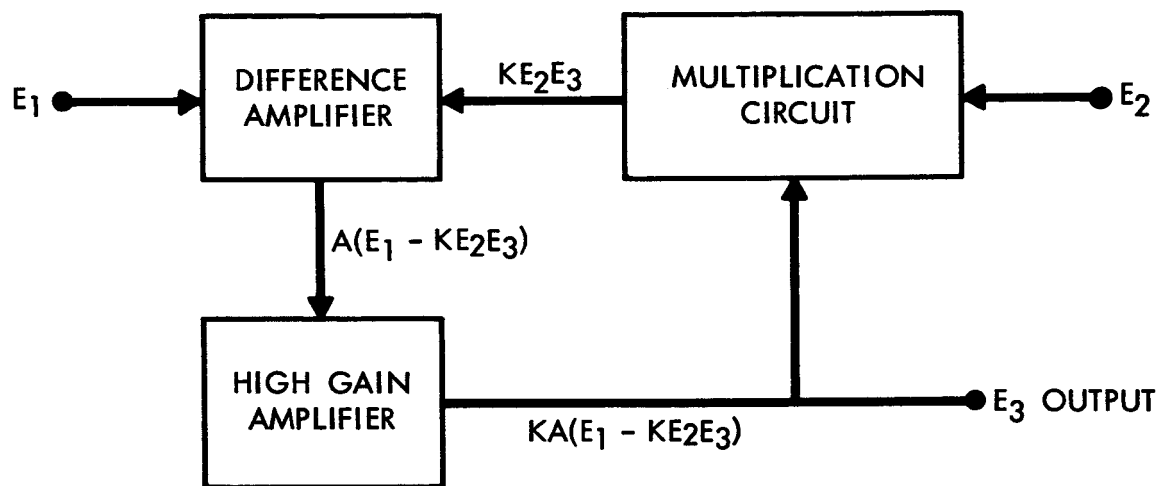


Figure 38. Horizontal Vibration Setup



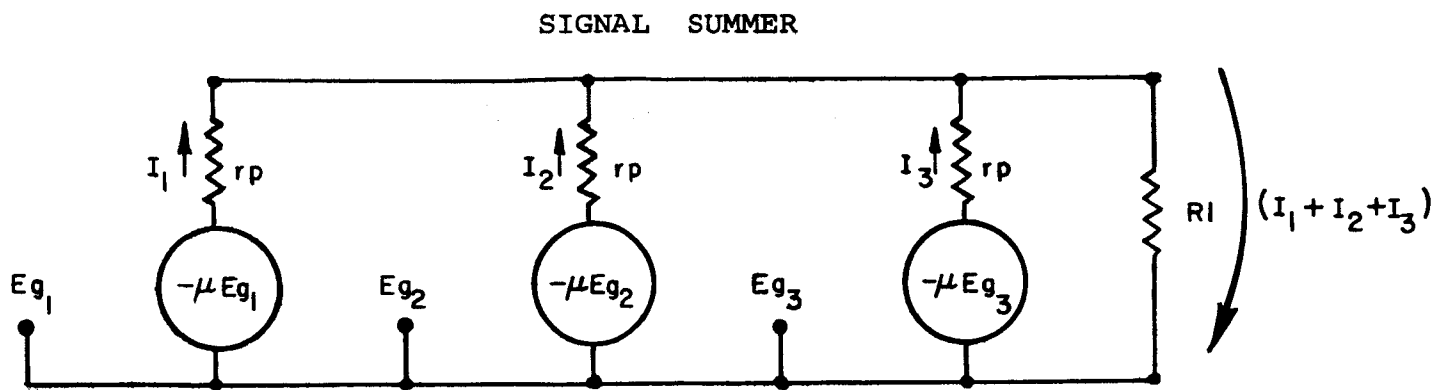
$$E_3 = KA(E_1 - KE_2E_3)$$

BECAUSE THE AMPLIFIER HAS HIGH GAIN,

$$\frac{E_3}{KA} \approx 0$$

THEREFORE:  $E_1 - KE_2E_3 \approx 0$  AND  $E_3 \approx K \frac{E_1}{E_2}$

Figure 39. Ratio Computer



Consider three identical triode amplifiers which can be symbolized as voltage generators of internal impedance  $r_p$ . The mesh equations are:

$$-\mu E_{g_1} = I_1 (r_p + R_l) + I_2 R_l + I_3 R_l$$

$$-\mu E_{g_2} = I_2 (r_p + R_l) + I_1 R_l + I_3 R_l$$

$$-\mu E_{g_3} = I_3 (r_p + R_l) + I_1 R_l + I_2 R_l$$

If all three stages are identical:

$$-3\mu E_{g_1} = 3I_1 r_p + 3I_1 R_l + 3I_1 R_l + 3I_1 R_l$$

$$I_1 (r_p + 3R_l) = \mu E_{g_1}$$

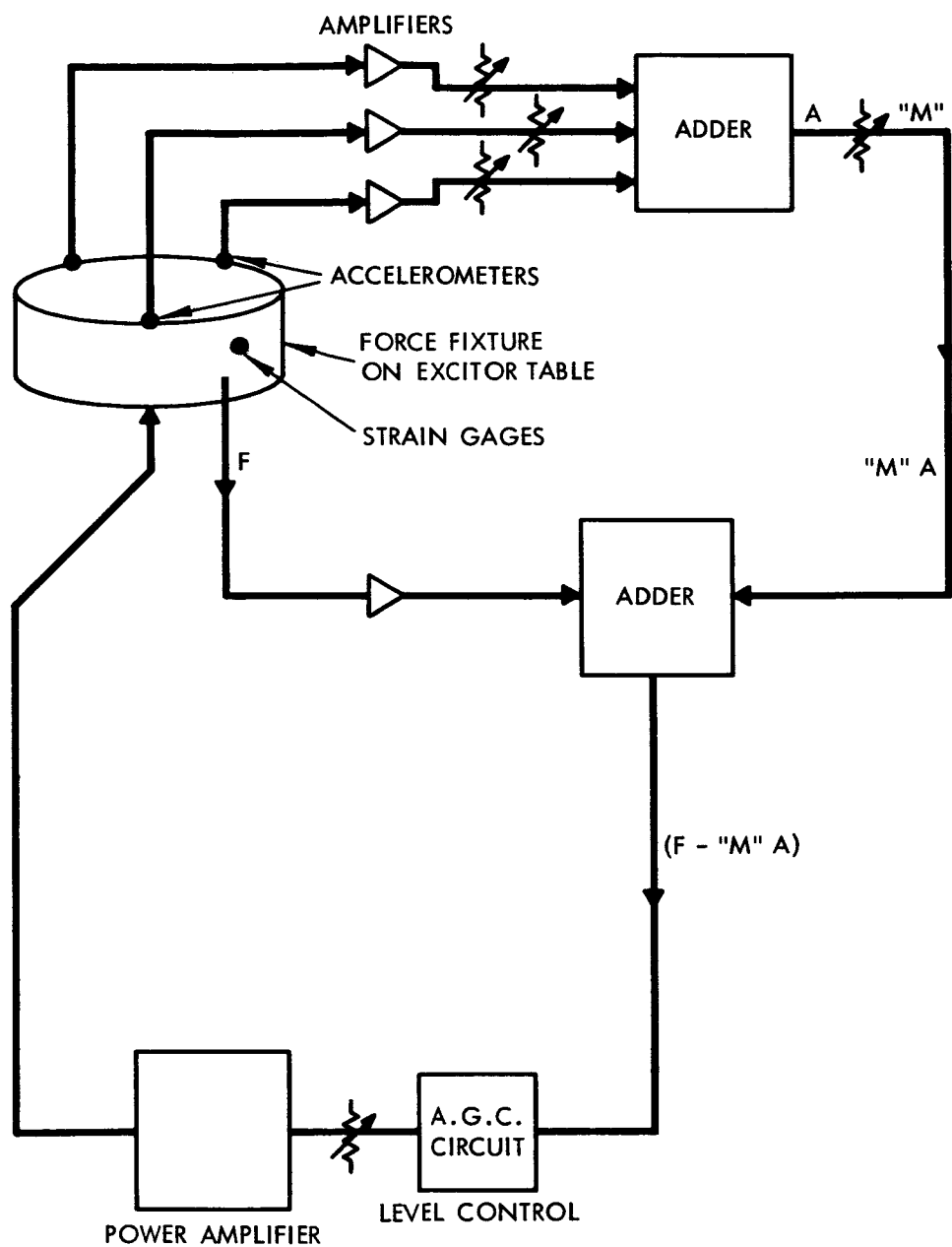
$$I_1 = \frac{-\mu E_{g_1}}{3R_l + r_p}$$

$$E_{out} = 3I_1 R_l = \frac{-3\mu E_{g_1} R_l}{3R_l + r_p}$$

Similarly, for  $n$  identical stages:

$$E_{out} = \frac{-\mu}{n + \frac{r_p}{R_l}} \sum_n (E_g)$$





A Force Programing Servo System

## RANDOM SIGNAL ANALYSIS

The differences between a periodic signal voltage and a random signal voltage can be graphically described by plotting hypothetical samples of each with respect to time as in Figure 1. Further, plots of acceleration, and spectral density as in Figure 2 will help to illustrate some of the properties of both types of signals. It is obvious from the figure that, although periodic signals can be measured in terms of acceleration, random signals are best treated in terms of acceleration density.

The outstanding feature of a periodic signal is that it can readily be described by a simple periodic function or a Fourier series of periodic functions and that the amplitude of the signal is entirely predictable as a function of time. A random signal presents some difficulties in this respect. By "random" we mean that the amplitude of the signal can theoretically be any value. "Gaussian" means that the amplitude distribution follows the normal curve of statistical analysis. This is generally the type of distribution referred to as "random noise" and is the only type of distribution to be discussed here. "White noise" refers to a constant spectral density over a given bandwidth. The "spectral density" of a random signal can be defined as:

$$G = \lim_{B \rightarrow 0} \frac{a^2}{B}$$

where: B is frequency bandwidth  $f_2 - f_1$  and a is the rms acceleration.

Thus:

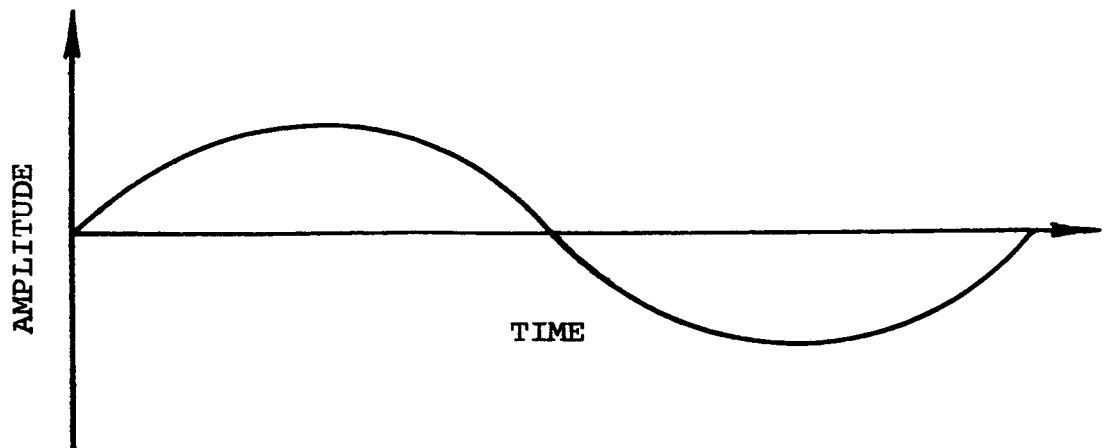
$$a^2 = \int_{f_1}^{f_2} G(f) df$$

Which reduces to  $a = (G B)^{\frac{1}{2}}$  for bandlimited white Gaussian noise. The term "Power Spectral Density" refers to plots of velocity distributions versus frequency.

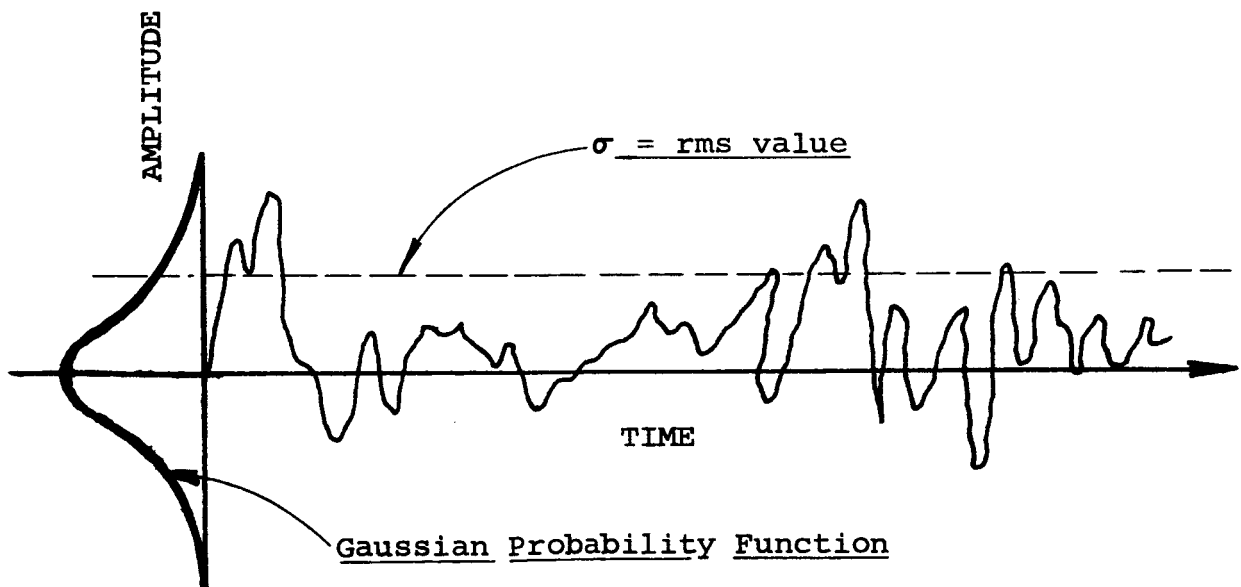
With reference to the Normal Curve of statistics, one can say that 68% of the time the signal amplitude is less than the rms value, and that 97% of the time the amplitude is less than three times the rms value. The rms value referred to is derived from the heating effect on a resistor caused by a randomly fluctuating current.

Enclosure 3

Comparison Between Periodic and Random Signals



Future Amplitude is Predictable



Future Amplitude Not Predictable

Figure 1



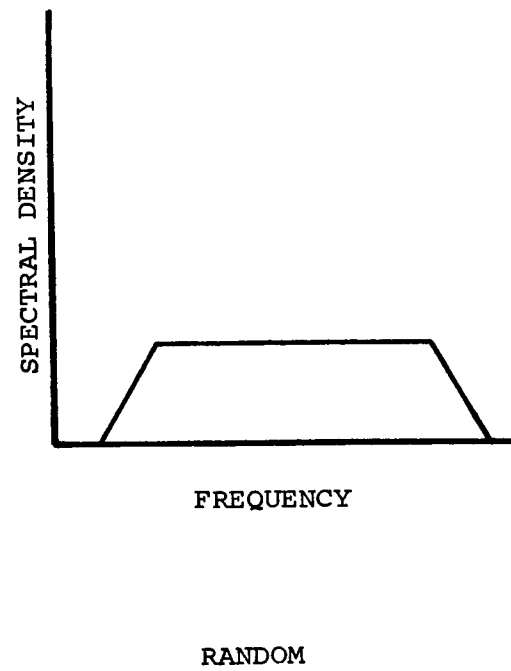
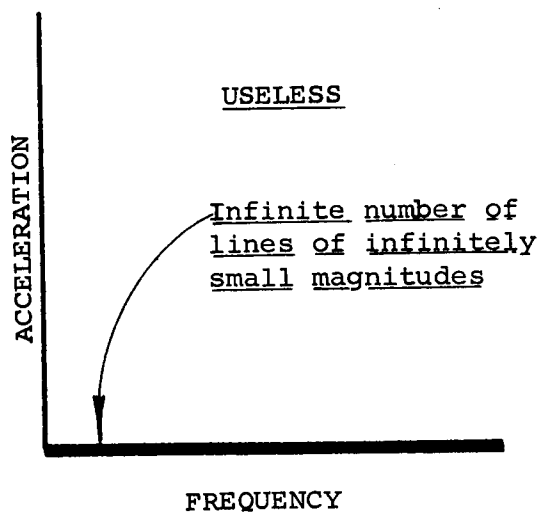
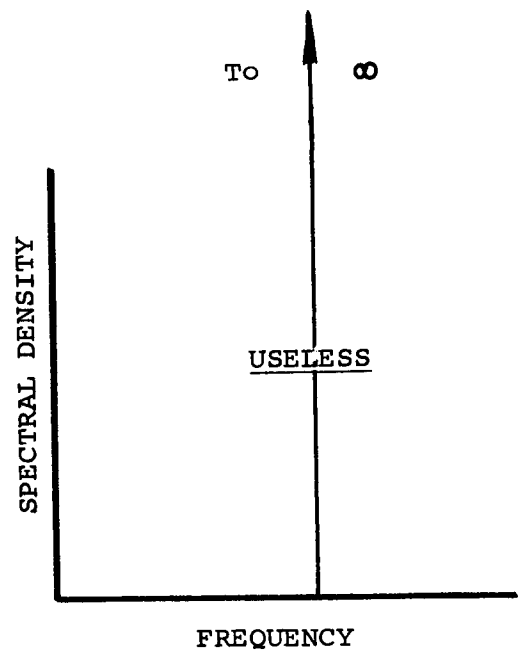
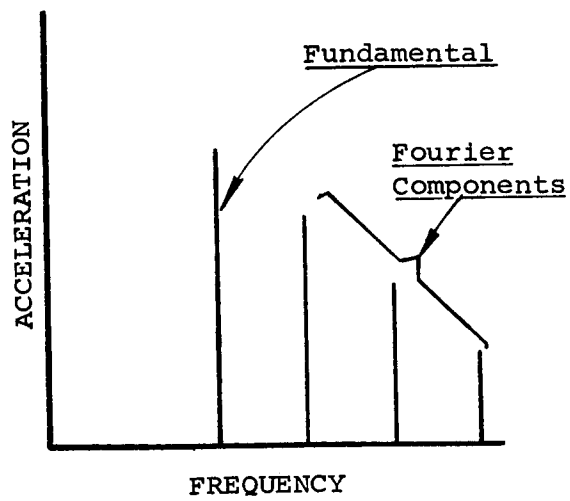


Figure 2

Of particular importance in random signal analysis is the proper selection of the equipment employed to measure random voltages. To explain why one must derive the relationships between "peak", "average", and "rms" values of a sinusoidal signal and compare one's results with corresponding values for "random".

An "rms" value of a sinusoidal function is defined in terms of the average power dissipation of a sinusoidal current in a resistance. If  $i$  is the instantaneous current, and  $I$  is the maximum value;

$$P = i^2 R$$

$i^2 = I^2 \sin^2 x$  which is a sinusoidal function that never crosses the abscissa. The average value of  $I^2 \sin^2 x$  is  $\frac{I^2}{2} = (\bar{i})^2$

The Voltage corresponding to the average current is:

$$V = (\bar{i}) R = \left( \frac{I^2}{2} \right)^{\frac{1}{2}} R = \frac{IR}{\sqrt{2}} = V_{\max} \times 0.707 = \text{rms value.}$$

The "average" value of a sinusoidal function can be found by dividing the area under the curve by the length of a half period:

$$\int_0^\pi \frac{V \sin x \, dx}{\pi} = \frac{2V}{\pi} = 0.637 \, V_{\max}.$$

The ratio of "rms" to "average" for a sinusoidal function is 1.11. The ratio for white Gaussian noise is 1.25. Thus, an ordinary averaging type a.c. vacuum tube voltmeter such as the Ballantine Model 300 or Hewlett Packard Model 1051 is made to read the "rms" value of a sinusoidal voltage through the use of a scale factor of 1.11 and can be made to indicate the true "rms" of a white Gaussian noise signal. In the latter case the meter reading is multiplied by the scale factor ratio:

$$\frac{1.25}{1.11} = 1.13$$

Ordinarily, due to limitations of meter amplifier dynamic range, the necessity of adding a 3,000 ufd. time-averaging capacitor across the meter terminals, and the limitations with respect to the types of random signal which can be measured, it is better to use a thermocouple type voltmeter or a special "square law" detector type of instrument such as the Ballantine Model 320. The Ballantine meter has an advantage over thermocouple instruments because thermocouples burn out when overloaded, whereas the circuitry of the Model 320 voltmeter, as is true also with most A.C. vacuum-tube voltmeters, is designed in such a way that the meter amplifier will limit the current flow to the indicator well below the burnout point. The Model 320 employs a special segmented diode circuit to simulate the parabolic curve desired. It is important to remember that this circuit is a power law device rather than square law and therefore can, under certain conditions, lead to inaccuracies.

## Required Loop Length for Random Signal Analysis

One problem it is necessary to solve if one is to make an accurate analysis of a random signal via the tape recorder-tape loop method concerns the length of the loop. The derivation of the minimum length for reasonably accurate analysis as enclosed here is made with the assumption that the standard error of the spectral density is proportional to the standard error of the mean square value of white noise passing through a single degree of freedom system. This is true if the mean square value of the output of the filter is divided by the bandwidth of the filter. Note the definition:

$$\text{Spectral density} = G = \text{limit} \frac{(\text{rms acceleration})^2}{(\text{noise bandwidth})}$$

$$Se = \sqrt{\frac{2}{2\pi T f \delta}} \times 100 \text{ C \%}$$

Where:

Se = standard error of mean square value of white noise passing through a linear single degree of freedom system.

T = integration time of the squared output.

f = center frequency of the filter.

$\delta = \frac{\text{Absolute Bandwidth of Filter}}{\text{Center frequency of filter}} = B = \text{relative bandwidth}$   
 $\frac{B}{f}$

Enclosure 4:

C is a correction factor defined by:

$$C^2 = 1 - \frac{1}{2\pi T f \delta} \left[ 1 - e^{-2\pi T f \delta} - \left(\frac{2\delta}{4-\delta}\right) e^{-2\pi T f \delta} \sin^2 \left( \pi T f \delta \sqrt{\frac{4}{\delta} - 1} \right) \right]$$

The sin term can be eliminated if  $\delta$  is small. If  $(T f \delta) = 8$ ,  $C \approx 0.99$ . For larger  $(T f \delta)$ , C becomes closer to 1.

$$\therefore Se \approx \frac{56.5}{\sqrt{T f \delta}} \%$$

It has been found that  $\delta \approx f^{-1/3}$ , gives a good compromise between the choices of a long analysis time--unnecessarily detailed analysis, and short analysis time--lack of enough detail.

If it is decided that  $Se = 40 f^{-1/3} \%$

then  $Se = 18.5\%$  at 10 cps

and  $Se = 10.0\%$  at 64 cps

$Se = 4.0\%$  at 1000 cps

This value of Se has been found to correspond well with the errors involved in acceleration sensing, recording and playback. The length of the tape loop then becomes:

$$T = \frac{56.5^2}{Se^2 f \delta} = \frac{3190}{1600 f^{-\frac{2}{3}} \times f \times f^{-\frac{1}{3}}} \approx 2 \text{ seconds}$$

Notice that, by choosing the standard error function carefully, the loop length becomes independent of frequency. A more detailed discussion of this and other data analysis problems is presented in Reference (6).

## Multi-Band Pass Filter Analyzer - Equalizer

The random signals recorded during the P-21 vibration tests were fed into the MB 80 bandpass filter random voltage analyzer system (Enclosure - Figure 1). The relative merits of the system can be compared by noticing the close agreement between the multifilter plot in Figure (24) and a plot of response amplitudes made with a sweep-frequency bandpass-filter analyzer. The Technical Products Analyzer incorporates a 5 cycle filter swept slowly across the frequency band of interest.

In order to determine how accurate the resulting curve in Figure (23) is and by comparison, how good the multifilter plot is, we must determine what sweeping rate will result in a negligible analyzer error, and what length the tape recorder loop must be to constitute a fair sample of the entire random test. These two items are the chief criteria for an accurate analysis. The shortest sample time for a tape loop analysis of a random signal of 2000 cps bandwidth which gives a standard error in the same range as that developed in data recording and playback, is in the order of two seconds. The scanning rate which will allow the energy within the filter to reach a nearly steady-state condition before the filter has been moved by 25% of its bandwidth can be expressed:  $\text{cps/s} = \left(\frac{B}{2}\right)^2$  where B is the

bandwidth of the scanning filter. For a 5 cycle wide filter and a 2,000 cycle wide spectrum, the scanning time should be at least 5 1/2 minutes. This requirement is satisfied by the TP Analyzer employed in making the plot of Figure (23). The time factor is considerable if a detailed picture of a random signal is desired. A 2,000 cycle bandwidth signal carefully analyzed via a 2 cycle wide filter would necessitate an analysis time approaching two hours or more. Thus, we have shown that the 80 second graphs derived from the multifilter system provide a time advantage and, though lacking in some details, agree within a few db with those obtained via the Technical Products Analyzer.

The reason that the 80 channel filter system can be calibrated to read directly in  $g^2/\text{cps}$  without a squaring operation lies in the fact that the output of a filter which is narrow with respect to the bandwidth of the signal fed into the filter is Gaussian regardless of the nature of the signal input. (See Ref. 4). Mr. S. P. Lloyd (Reference 7) indicated that there are conditions where the signal output would be less Gaussian than the input, but such occurrences should be rare.

Enclosure 5

The multifilter analyzer concept can be extended in the form of an automatic equalizer-analyzer control for a vibration exciter system. The Structural Dynamics Branch will soon have such a system in operation. The chief benefit of having the automatic equalizer-analyzer will be the time saving. Equalization time will be cut from hours to seconds, even for most complex spacecraft structures. The operation of the system can readily be understood by referring to Enclosure - Figure 2 and comparing it with Enclosure - Figure 3.

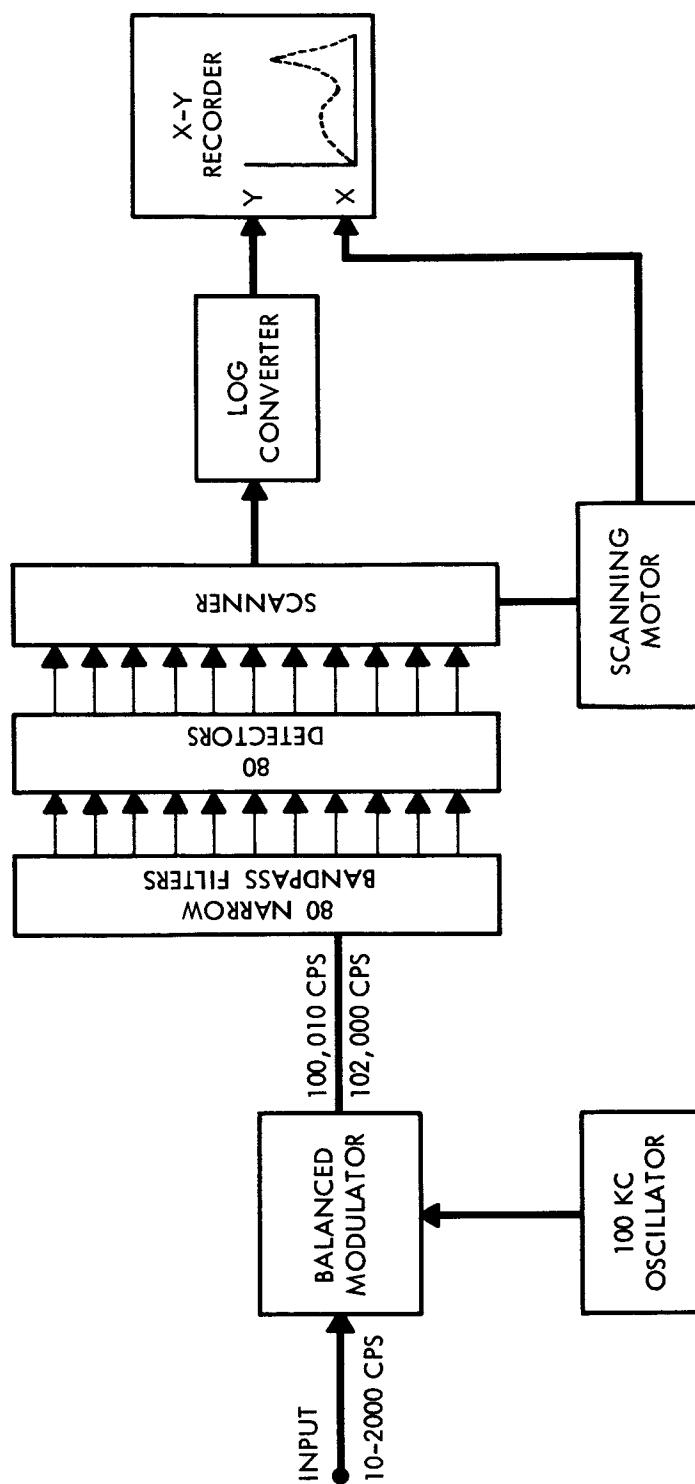


Figure 1. Multi-Filter Analyzer System



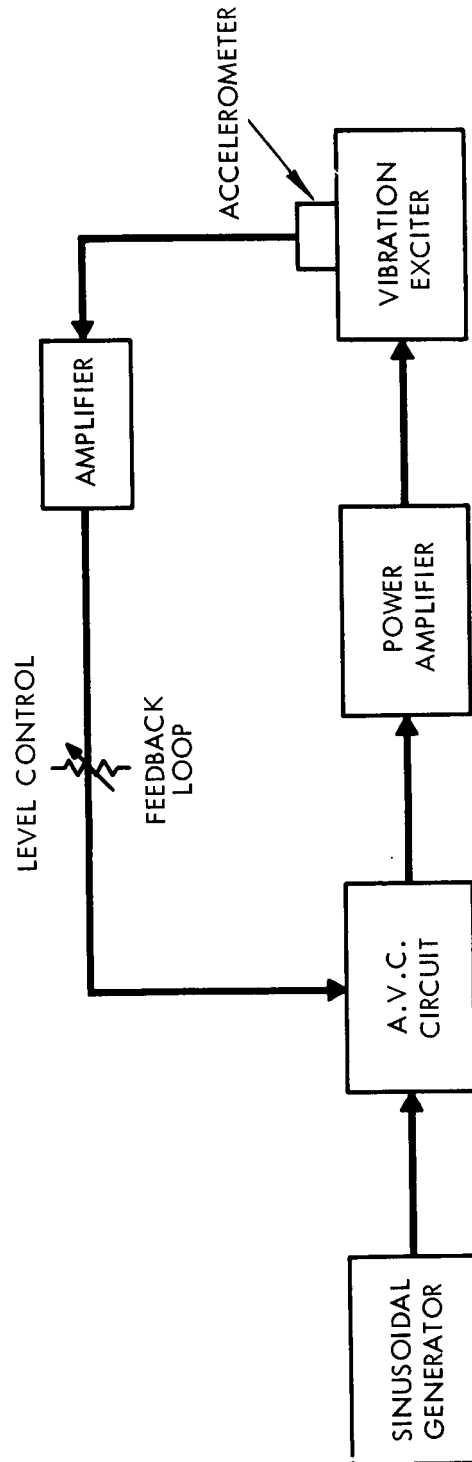


Figure 2. Sinusoidal Servo Control

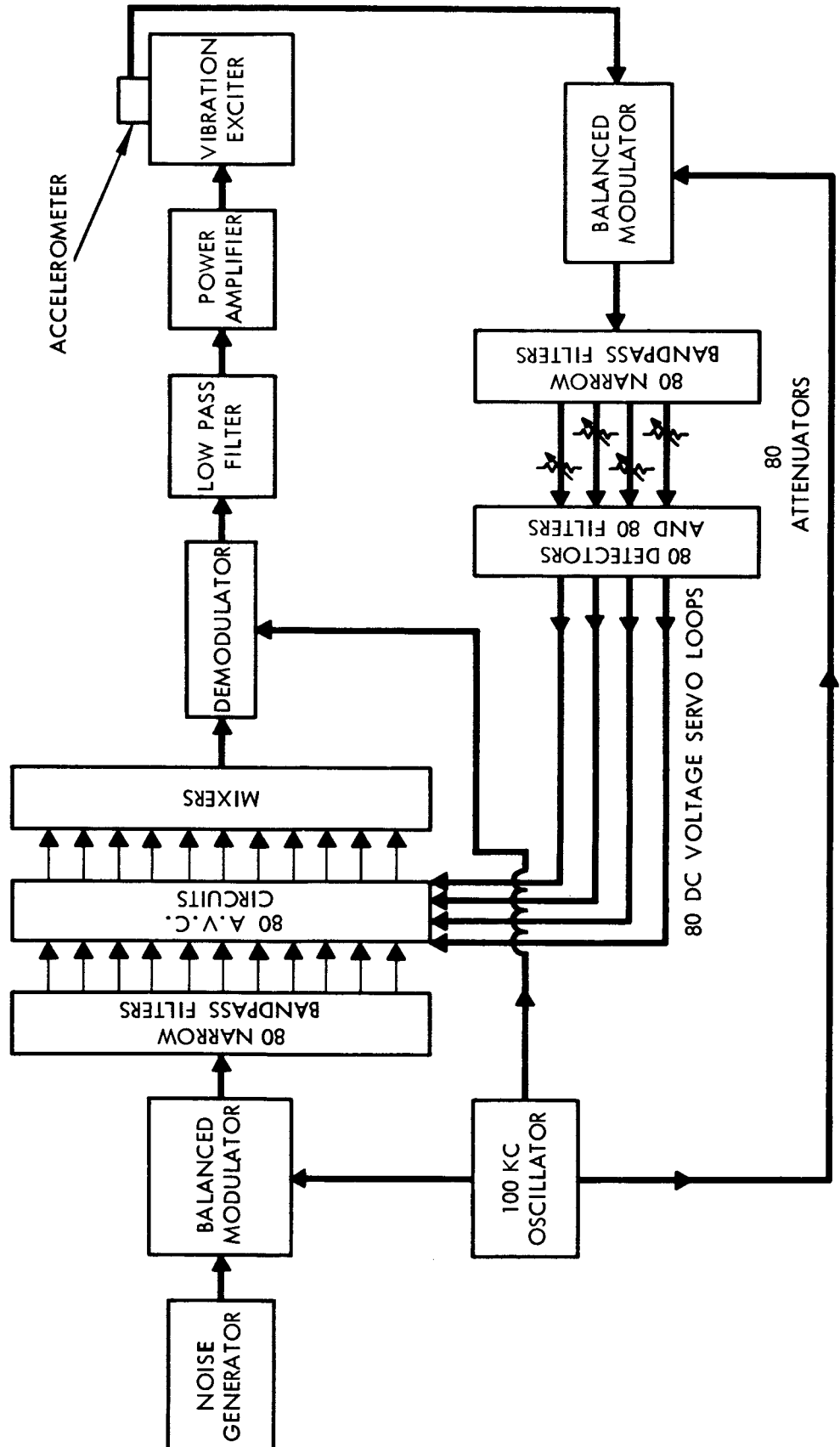


Figure 3. Automatic Random Equalizer

### Frequency Sweep Schedule

<u>Vibration</u> <u>Axis</u>	<u>Frequency</u> <u>Range</u> <u>cps</u>	<u>Test</u> <u>Duration</u> <u>- min.</u>	<u>Acceleration</u> <u>g, 0-to-peak</u>
Thrust (z-z axis)	5-50	0.83	1.5 (a)
	50-500	0.83	7.1
	500-2000	0.50	14.
	2000-3000	0.15	36.
	3000-5000	<u>0.18</u>	14. (b)
Total -		2.5 min.	
Lateral (x-x axis) & Lateral (y-y axis)	5-50	0.83	0.6 (a)
	50-500	0.83	1.4
	500-2000	0.50	2.8
	2000-5000	<u>0.33</u>	11.3 (b)
Total -		2.5 min.	
<u>Grand Total -</u>		<u>7.5 min.</u>	

Note (a) Within maximum amplitude limit of vibration generator.

Note (b) Within maximum frequency limit of vibration generator.

2.2 Random Motion Vibration. Gaussian random vibration shall be applied by shaping the input according to the test schedule given below. Transition from one PSD to the other shall be made using a filter whose characteristic rolloff is at a rate of approximately 12 db/octave. Peak-notch equalization, with spacecraft installed, at specified PSD shall be within  $\pm 3$  db. Rolloff characteristic above 2000 cps shall be at a rate of 40 db/octave or greater.

### Random Vibration Schedule

<u>Vibration</u> <u>Axis</u>	<u>Frequency</u> <u>Range</u> <u>cps</u>	<u>Test</u> <u>Duration</u> <u>min.</u>	<u>PSD</u> <u>Level</u> <u>g<sup>2</sup>/cps</u>	<u>Approx.</u> <u>Accel.</u> <u>g-rms</u>
Thrust (z-z axis)	5-200	2	0.06	3.4 *
	200-400		See Par. 2.2	1.7
	400-2000		0.005	<u>2.8</u>
	Total			5.3

\* Within maximum amplitude limit of vibration generator.

## Random Vibration Schedule-Cont'd

<u>Vibration</u> <u>Axis</u>	<u>Frequency</u> <u>Range</u> <u>cps</u>	<u>Test</u> <u>Duration</u> <u>min.</u>	<u>PSD</u> <u>Level</u> <u>g<sup>2</sup>/cps</u>	<u>Approx.</u> <u>Accel.</u> <u>g-rms</u>
Lateral	5-25		0.30	2.5 *
(x-x axis)	25-100	2	See Par. 2.2	1.2
&	100-2000	(each axis)	0.005	3.1
Lateral				
(y-y axis)				Total 4.2
Total 6 min.				

2.3 Combustion Resonance Dwell This test simulates a measured combustion oscillation condition observed in the X-248 solid-propellant rocket motor. The range of the test is from 550 to 650 cps. The test is conducted by traversing the 100 cps wide band slowly such that 1/4 minute is consumed in moving from 550 to 650 cps. Rate of change of frequency with time shall be proportional to frequency.

2.3.1 Apparent Weight To determine control acceleration to be used for this test, the apparent weight of the spacecraft in the vicinity of 600 cps may be determined by measurement. This will require that both force and acceleration be measured at a point near the spacecraft-jig interface, or the apparent weight may be deduced at this point by measurement elsewhere. Control acceleration then is computed by dividing  $\pm 400$  lbs. force (thrust axis) or  $\pm 50$  lbs. force (lateral axes) by the apparent weight determined -- from the formula: apparent weight = force/g-acceleration (resolved at a point near the jig-spacecraft interface). The apparent weight should be averaged from measurements made over the 550-650 cps range.

2.3.2 Force Programming A superior alternate method may be substituted wherein vibration force is programmed by servoloop control of a suitable jig such that  $\pm 50$  lbs. force (lateral) may be applied at the point where the spacecraft is installed. If the jig is properly calibrated, and control signals compensated for jig-driving force, apparent weight of the spacecraft need not be determined.

2.3.3 Assumed Apparent Weight A second alternate method may be employed wherein an apparent weight of the spacecraft is assumed to be seven pounds. With this assumption, the control acceleration should be 57 g (0-to-peak) for thrust axis vibration, and 7.1 g (0-to-peak) for each of the two lateral axes.

\* Within maximum amplitude of vibration generator.

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